Design and Optimization of Megahertz Wireless Power Transfer Systems

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Outline

- Introduction
- Parameter Design
- Harmonic Reduction
- Robust Design
- Matching Network Design
- Other Ongoing Activities
- Conclusion
Shanghai Jiao Tong University  Dec. 2015

- 28 Schools/Departments
- 13 Affiliated Hospitals
- 16,188 Undergraduates
- 28,842 Graduates (≈64%)
  - 6,506 Ph.D. students
- 2,793 Faculties
  - 890 Professors
- 3.3km² (Minhang Campus)
1. Battery / Energy Management
2. Wireless Power Transfer
3. Electric Vehicle Dynamics
4. Servo/Motion Control

Dynamic Systems Control Laboratory, UM-SJTU Joint Institute
Initial Efforts Starting from 2010

<table>
<thead>
<tr>
<th>Gap (cm)</th>
<th>5.6</th>
<th>10.1</th>
<th>14.8</th>
<th>19.3</th>
<th>24.1</th>
<th>28</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency (%)</td>
<td>88.84</td>
<td>93.32</td>
<td><strong>93.69</strong></td>
<td>92.53</td>
<td>88.07</td>
<td>70.04</td>
</tr>
<tr>
<td>$F_m$ (MHz)</td>
<td>13.59</td>
<td>14.74</td>
<td>15.27</td>
<td>15.71</td>
<td>16.11</td>
<td>16.08</td>
</tr>
<tr>
<td>$F_e$ (MHz)</td>
<td>19.87</td>
<td>17.85</td>
<td>17.01</td>
<td>16.51</td>
<td>16.11</td>
<td>16.08</td>
</tr>
</tbody>
</table>

Multimeters for voltage and current measurement

DC link power supply

Supercapacitor module + Rectifier

Resonating coil

Resonating coil

Resonating coil

FPGA board

MOSFET driver IC

1MHz PWM input signal generation FPGA board

Resonant Inverter

Vehicle track

Emitting coil (T1)

Repeating coil (T2)

Repeating coil (T3)

Receiving coil (T4)

High frequency rectifier

Supercapacitor module
System-level Design, Optimization, and Control

- Optimal load analysis and tracking
- Optimal and robust designs of system parameters
- Design and power flow control in multi-receiver systems

Working frequency: 6.78MHz
Power level: 20 W
System Efficiency: 84% ($k=0.1327$)
Major Challenges in MHz WPT

- More obvious nonlinearities of the devices and thus non-neglectable reactance
- Potentially higher switching loss and thus lower system **Efficiency**
- More challenging Electromagnetic interference (**EMI**) problem
- **Robustness** again varying operation condition (i.e., coupling and load)

Keywords: MHz wireless power transfer, high efficiency, low-harmonic contents, robustness
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Conventional Design

System Configuration

- Power Supply \( P_{in} \) → Class E PA \( P_{zin} \) → Coupling Coils → Full Bridge Rectifier \( P_{rec} \) → DC Load \( P_{o} \)

Conventional Design

- Input reactance of the full-bridge rectifier is completely neglected;
- The compensation capacitors are designed to resonant with coupling coils;
- The Class E PA is optimized based on the input impedance of coupling coils.

Problems

- Large switching loss on the full-bridge rectifier at MHz;
- Difficult to analytical derive the input reactance of the rectifier;
- Non-zero rectifier input reactance detunes the coupling coils from resonance;
- It also cause the PA to deviate from its ideal ZVS operation.
Select a high-efficiency rectifying circuit;
Derive an analytical expression of the input impedance of the rectifier;
Design parameters based on the derived input impedance of the rectifier.
Rectifier Input Impedance

- The analytically derived input impedance of the Class E rectifier and the relationship between $C_r$ and $D$.

\[ X_{rec} = \frac{V_{m}X_{rec}}{I_{m}} = -\frac{1}{\pi} \left[ a + \frac{b}{\omega C_r} + r_{D_r} (c + d) \right] \]

\[ R_{rec} = 2\sin^2 \phi_{rec} (R_L + r_{L_r}) + 2\varepsilon r_{D_r} \]

\[ C_r = \frac{1 + \left[ \sin 2\pi D + 2\pi (1-D) \right]^2}{1 - \cos 2\pi D} - \frac{2\pi^2 (1-D)^2 - \cos 2\pi D}{2\pi \omega (R_L + r_{L_r} + r_{D_r})} \]
System-Level Optimization

- **Rectifier**: $C_r$ that enables a 0.5 duty cycle, $D$;
- **Receiving coil**: $C_{rx}$ that makes the coupling coils truly resonant;
- **PA**: $C_S$ that follows the Raab’s equations and the load of PA.
Optimized Parameter Design

\[ C_{r,\text{opt}} = \frac{747.2}{R_L + r_{L_T} + r_{D_T}} \times 10^{-12} \]

\[ \eta_{\text{opt}} = \frac{R_L}{R_L + r_{L_T} + 1.2337r_{D_T}} \]

\[ C_{r,x,\text{opt}} = \frac{1}{\omega \left[ \omega L_{r,x} - (0.6648(R_L + R_{L_T}) + 0.8484r_{D_T}) \right]} \]

\[ \eta_{\text{coil}} = r_{r,x} [r_{r,x} + 0.5768(R_T + r_I) + 0.7116r_{R_N}] + \omega^2 L^2_{m} \]

\[ C_{S,\text{opt}} = \frac{0.1836[r_{r,x} + 0.5768(R_L + R_{L_T}) + 0.7116r_{D_T}]}{\omega r_{r,x} [r_{r,x} + 0.5768(R_L + R_{L_T}) + 0.7116r_{D_T}] + \omega^3 L^2_m} \]

\[ X_{0,\text{opt}} = 1.1525 r_{r,x} + \frac{1.1525 \omega^2 k^2 L_{r,x} L_{r,x}}{r_{r,x} + 0.5768(R_L + R_{L_T}) + 0.7116r_{D_T}} \]

\[ \eta_{\text{PA}} = \frac{P_{\text{Zin}}}{P_{\text{in}}} = \frac{g^2 R_{Zin}}{2R_{dc} + 2r_{L_T} + g^2 r_{L_O}} \]

\[ g = 2\pi \sin(\varphi + \phi) + 4\cos(\varphi + \phi) \]

\[ \varphi = \arctan \frac{X_0}{R_{Zin}} \]

\[ \phi = \arctan \frac{\frac{k^2}{g^2} - 4 - \pi \omega C_S (2R_{Zin} + \pi X_0)}{\pi + \pi \omega C_S R_{Zin} - 2\pi \omega C_S X_0} \]
Results (6.78MHz, $k=0.1327$, 84%)
Analytically derived characteristics of the circuits, particularly the input impedance of the rectifier;

A system-level approach starting from rectifier and being extended to optimize the coupling coils and PA.

A very high dc-to-dc system efficiency, 84%, is achieved with loosely coupled coils, $k=0.1327$ (40mm).

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Motivation

- EMI improvement through optimized design of circuits in a MHz WPT system;
- Reduction of THD of the input/output voltage of coupling coils;
- A high system efficiency at the same time.

Due to the series-series compensation, the input and output currents of the coupling coils are sinusoidal. Thus the THDs of their input/output voltages are criteria to verify the improvement on EMI.
Class-E Full-Wave Rectifier

- A promising candidate because of its sinusoidal input voltage and current.
- A 0.49 duty cycle of the rectifying diodes that avoids the overlapping and maximizes the power output capability of the rectifier.

\[
C = \frac{1}{\omega R_L} \left[ \frac{1}{4\pi} - \frac{\pi}{2} (1 - D)^2 \right] + \frac{2\pi (1 - D) \cos(\phi_{rec} + 2\pi D) - \sin(\phi_{rec})}{4\pi \sin(\phi_{rec} + 2\pi D)}
\]

\[
C_{opt} = \frac{0.1756}{\omega R_L,\text{min}}
\]
The other parameters are designed following the procedures previously explained.

The THDs are compared with those of the conventional full-bridge rectifier.

Results - THD with $D=0.49$

- Full-Wave (76.49%↓)
- Full-Bridge (54.8%↓)
- (14.6%↓)
- (22.0%↓)
Results - Power Losses

Loss breakdown (10 W, 30 Ω $R_L$)

<table>
<thead>
<tr>
<th>Loss</th>
<th>Full-wave Rec.</th>
<th>Full-bridge Rec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{sw}$</td>
<td>0.26 W</td>
<td>0.61 W</td>
</tr>
<tr>
<td>$P_{cd}$</td>
<td>0.29 W</td>
<td>0.60 W</td>
</tr>
<tr>
<td>$P_L$</td>
<td>0.03 W</td>
<td>-</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
P_{cd} &= \frac{1}{2\pi} \left\{ \int_0^{2\pi D} \left[ V_F i_{D_1}(\omega t) + r_D i_{D_1}^2(\omega t) \right] d\omega t \\
& \quad + \int_{2\pi(1-D)}^{2\pi} \left[ V_F i_{D_2}(\omega t) + r_D i_{D_2}^2(\omega t) \right] d\omega t \right\}, \\

P_L &= 2 \left( \frac{I_o}{2} \right)^2 r_L \\
P_{sw} &= P_{rec} - P_o - P_L - P_{cd}.
\end{align*}
\]
Results—(80% Ave. Efficiency)

Loss breakdown (10 W system input power, 30 Ω \( R_L \))

<table>
<thead>
<tr>
<th>Loss</th>
<th>WPT system (Full-wave Rec.)</th>
<th>WPT system (Full-bridge Rec.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectifier</td>
<td>0.55 W</td>
<td>1.06 W</td>
</tr>
<tr>
<td>Coupling Coils</td>
<td>0.33 W</td>
<td>0.55 W</td>
</tr>
<tr>
<td>PA</td>
<td>0.42 W</td>
<td>0.65 W</td>
</tr>
<tr>
<td>Total</td>
<td>1.30 W</td>
<td>2.26 W</td>
</tr>
</tbody>
</table>

Power losses from the rectifiers significantly influence the overall efficiencies (42.48%↓).

Under different loads

Under different coupling
The Class-E full-wave rectifier is proposed to improve the EMI problem;

A systematic design approach is developed to design the rectifier, coupling coils, and PA;

Significant THD reduction, 76.49%, in input voltage of the rectifier is achieved comparing with that in the conventional full-bridge rectifier.

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Most of existing designs target on a single fixed operating condition, i.e., fixed coil relative position and load.

However, in real applications changes in the coil relative position and final dc load are common.

A design methodology, active or passive, is required to optimize the performance over the possible ranges of the coil relative position and load.
Optimal Load for High Efficiency

Optimal loads

- PA
- Rectifier
- DC/DC converter
- Load

$P_f$, $L_m$, $R_L$, $P_L$
Improved Charging Efficiency

- Wireless charging efficiency improvement with a fixed coil relative position.

The experimental WPT system. (a) Overall system. (b) Relative position of coils. (c) Power sensor. (d) I/V sampling board. (e) Cascaded DC/DC converter.
Hill-climbing Tracking of Optimal Load

A varying load resistance

A varying coil position

Fig. 1 Tracking of optimal load resistances with a varying $R_l$.

Fig. 2 Tracking of optimal load resistances with a varying $k$.

Again, the system efficiency of the MHz Class E² WPT system is analytically derived.

\[
\eta_{sys} = \eta_{pa} \cdot \eta_{coil} \cdot \eta_{rec} = \frac{P_o}{P_{in}}
\]

\[
\eta_{pa} = \frac{P_{Z_{in}}}{P_{in}}\quad \eta_{coil} = \frac{P_{rec}}{P_{Z_{in}}}\quad \text{and} \quad \eta_{rec} = \frac{P_o}{P_{rec}}
\]

\[
\eta_{pa} = \frac{P_{Z_{in}}}{P_{in}} = \frac{a^2(R_0 - r_{L0})}{2R_{dc} + 2r_{Lf} + a^2r_{L0} + (1 + \frac{2a}{\pi} + \frac{a^2}{2})r_Q}
\]

\[
\eta_{coil} = \frac{R_{rec}\omega^2k^2L_{tx}L_{rx}}{\omega^2k^2L_{tx}L_{rx}(R_{rec} + r_{rx}) + r_{tx}b}
\]

\[
\eta_{rec} = \frac{P_o}{P_{rec}} = \frac{R_L}{R_L + r_{Lr} + \frac{cr_{r}}{\sin^2\phi_{rec}}}
\]
Original Class E PA matching network has poor robustness.

Robustness Index

\[ \alpha_x = \max \left| \frac{\eta_x(k, R_L) - \eta_x(0.203, 30)}{\eta_x(0.203, 30)} \right| \]

<table>
<thead>
<tr>
<th>( \alpha_{pa} )</th>
<th>( \alpha_{coil} )</th>
<th>( \alpha_{rec} )</th>
<th>( \alpha_{sys} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>47.0%</td>
<td>5.3%</td>
<td>4.2%</td>
<td>47.6%</td>
</tr>
</tbody>
</table>

Note: A smaller \( \alpha \) corresponds to improved robustness.
Modified MN and Design Problem

Circuit Improvement

Robust Optimization

Definitions of Parameters

<table>
<thead>
<tr>
<th>Vector</th>
<th>Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$C_S, C_D, C_1, C_{rx}, C_T$</td>
</tr>
<tr>
<td>$P_{con}$</td>
<td>$R_{in}, R_{out}$</td>
</tr>
<tr>
<td>$P_{var}$</td>
<td>$C_{con}, R_{in}^v$</td>
</tr>
<tr>
<td>$P_{max}$</td>
<td>$p_{max}, p_{var}^v$</td>
</tr>
<tr>
<td>$D_{con}$</td>
<td>$\omega, C_{ls}, L_0, L_{ls}, L_{ps}, r_S, r_{lf}, r_{tg}, r_{fs}, r_{ta}, r_{ls}, r_{ps}$</td>
</tr>
</tbody>
</table>

Optimization Problem

$$\max_{x} \eta_{sys}^{\text{nom}} (x)$$

s.t. 
$$\alpha_{sys} (x) \leq \alpha_{sys}^\text{max},$$
$$\max_{P_{var}} |D (x, P_{con}, P_{var}) - 0.5| \leq \beta_{D}^\text{max}$$

$$\alpha_{sys} (x) = \max_{P_{var}} \left| \frac{\eta_{sys} (x, P_{var}) - \eta_{sys}^{\text{nom}} (x)}{\eta_{sys}^{\text{nom}} (x)} \right|$$
$$= \max_{P_{var}} \left| \frac{f (x, P_{con}, P_{var}) - f (x, P_{con}, P_{var}^{\text{nom}})}{f (x, P_{con}, P_{var}^{\text{nom}})} \right|$$
Results

\begin{table}
\centering
\begin{tabular}{|c|c|c|c|}
\hline
& $\alpha_{pa}$ & $\alpha_{coil}$ & $\alpha_{Rec}$ & $\alpha_{sys}$ \\
\hline
Robust design & 11.1\% & 3.3\% & 3.1\% & 12.4\% \\
Existing design & 43.3\% & 5.8\% & 4.2\% & 44.1\% \\
\hline
\end{tabular}
\end{table}
A new circuit design methodology is developed that optimizes the performance over ranges rather than single fixed operating condition.

The PA matching network is modified to enhance the robustness of the load-sensitive Class E PA.

The potential of the circuits is maximized through the optimization-based parameter design, i.e., a multi-disciplinary approach.

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Motivation

- Further reduction of harmonic contents in the input voltage of coupling coils;
- A stable performance under variations in coupling and final load;
- A robust multiple-receiver system driven by a constant-current-mode PA;
- Circuit design methodology to achieve 1) low EMI, 2) high efficiency, and 3) high output power.
Matching Network

- MN is included to transform and thus provide desired impedances as the PA load.
Target Region of PA load

- A common region for both high efficiency and high output power.
The high THD of the input voltage of the coupling coils is mostly caused by the 2\textsuperscript{nd}-order harmonic.

\begin{itemize}
  \item THD of Input Current: 3.87\%
  \item THD of Input Voltage: 47.8\%
  \item THD of Output Current: 3.04\%
  \item THD of Output Voltage: 9.28\%
\end{itemize}
Matching Network Design

**Harmonics Suppression**

\[
V_{Z_{in},m}^{(1)} = \left| Z_{in} \right| I_{Z_{in},m}^{(1)}
\]

\[
\frac{I_{0,m}^{(1)}^2}{2R_{0}^{(1)}} = \frac{I_{Z_{in},m}^{(1)}^2}{2} R_{Z_{in}}^{(1)}
\]

\[
V_{Z_{in},m}^{(1)} = \left| Z_{in} \right| I_{0,m}^{(1)} \sqrt{\frac{R_{0}^{(1)}}{R_{Z_{in}}^{(1)}}}
\]

\[
V_{Z_{in},m}^{(2)} = \left| Z_{in} \right| I_{0,m}^{(2)} \sqrt{\frac{R_{0}^{(2)}}{R_{Z_{in}}^{(2)}}}
\]

\[
\frac{V_{Z_{in},m}^{(2)}}{V_{Z_{in},m}^{(1)}} = \frac{Z_{in}^{(2)}}{Z_{in}^{(1)}} \frac{I_{0,m}^{(2)}}{I_{0,m}^{(1)}} \sqrt{\frac{R_{0}^{(1)}}{R_{Z_{in}}^{(1)}}} \sqrt{\frac{R_{0}^{(2)}}{R_{Z_{in}}^{(2)}}}
\]

A smaller ratio of \( R_{0}^{(2)} \) to \( R_{0}^{(1)} \) results in a lower second-order harmonic.

**Design Procedure**

Define the feasible ranges of \( C_L \) and \( C_R \):

\[
C_L \in (C_{L}^{lower}, C_{L}^{upper})
\]

\[
C_R \in (C_{R}^{lower}, C_{R}^{upper})
\]

Define a target region as a constraint:

\[
R_{0}^{lower} \leq R_{0}^{(1)}(k, C_L, C_R) \leq R_{0}^{upper}
\]

\[
X_{0}^{lower} \leq X_{0}^{(1)}(k, C_L, C_R) \leq X_{0}^{upper}
\]

Add the 2nd-order harmonic suppression as another constraint:

\[
R_{0}^{(2)}(k, C_L, C_R) \leq \lambda \cdot R_{0}^{(1)}(k, C_L, C_R)
\]

where \( \lambda \) is an index. A smaller \( \lambda \) leads to a smaller 2nd-order harmonic.

The candidate combinations of the two capacitors can be obtained by simply sweeping \( C_L \) and \( C_R \) within their feasible ranges if the calculated \( R_{0}^{(1)}, X_{0}^{(1)}, R_{0}^{(2)} \) meet the two constraints under the varying \( k \).
Results

- The efficiency and output power of both the PA and system are significantly improved over a wide range of $k$;
- The second-order harmonic and THD of the input voltage of coupling coils are obviously reduced, 81.9%.
The reduction of harmonic contents in the input voltage of the coupling coils is discussed.

A matching network is proposed, designed, and implemented.

A design methodology is developed to perform the parameter design of the circuits at a system level.

The effort helps to design robust multiple-receiver WPT systems.

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Ongoing Activities

- Compensation and power distribution in multiple-receiver WPT systems
- Load pull analysis for CCM Class E PA
- Charging profile based optimization of MHz wireless battery charger

Multiple-receiver MHz WPT system

Compensation and power distribution in multiple-receiver WPT systems

Charging profile based optimization of MHz wireless battery charger


Thank You

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