

Course Description

Vv557 Methods of Applied Mathematics II Advanced Methods for PDEs

Summer Term 2014

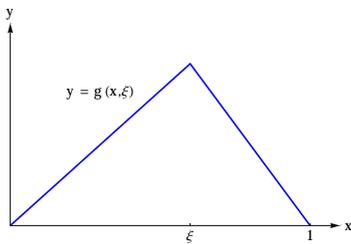


JOINT INSTITUTE
交大密西根学院

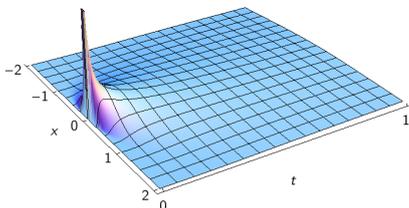
Prerequisites: Vv256 or Vv286 and the preceding calculus courses or graduate standing or permission of instructor.

Course website: <http://umji.sjtu.edu.cn/personal/horst/teaching/vv557.html>

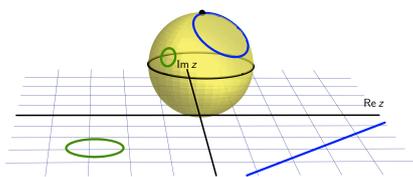
Intended Audience: ME and ECE undergraduate and graduate students. The course is accessible to sophomore students; no additional background is required. Basic knowledge of PDEs (e.g., as contained in *Vv454 Partial Differential Equations and Boundary Value Problems*) is very helpful.



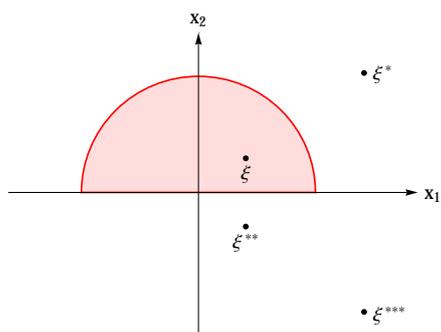
Non-Smooth Solutions to Differential Equations



Delta Families and Point Sources



Green functions via Conformal Maps



Green functions via Method of Images

Description: The course consists of three parts, each dealing with certain mathematical techniques useful for solving differential equations. Examples from mechanical as well as electrical engineering will be used throughout.

Our initial motivation is the desire to understand the treatment of point sources. Starting from the Dirac delta function as a formal symbol to denote a point source, we begin a formal treatment of generalized functions (distributions), including principal value integrals, notions of convergence and delta families, the distributional Fourier transform and solutions of distributional equations.

The second part of the course applies the theory of distributions to ordinary differential equations (ODEs). Strong, weak and distributional solutions are introduced and general solution formulas obtained. The main focus is then on obtaining Green's functions for boundary value problems (BVPs) for ODEs, leading to a brief discussion of solvability and modified Green's functions for ODEs.

The final third of the course extends the ODE methods to PDEs. Green's formulas for boundary value problems of the first, second and third kind are derived. Subsequently, methods for finding Green's functions are explored, including that of full and partial eigenfunction expansions, the method of images and (if time permits) conformal mappings. Finally, a short introduction to the use of Green's functions for the Laplace equation in the boundary element method (BEM) is presented.

Keywords: Point sources, distributions, classical, weak and fundamental solutions to differential equations, causal fundamental solutions, formal adjoint differential operator, conjunct, Green's formula, Green functions for boundary value problems for ODEs, modified Green functions for ODEs, solution formulas for ODEs and PDEs, Green functions for PDEs, eigenfunction expansions, method of images, conformal mappings, boundary element method.

Textbooks:

- Y. Pinchover and J. Rubinstein, *An Introduction to Partial Differential Equations*, Cambridge University Press 2005,
- I. Stakgold and M. Holst, *Green's Functions and Boundary Value Problems*, 3rd Ed., Wiley 2011,
- E. Zauderer, *Partial Differential Equations of Applied Mathematics*, 2nd Ed., Wiley 1989,

Syllabus:

Lecture	Lecture Subject
1	Introduction
2	Point Sources and Green Functions
3	Distributions
4	Operations on Distributions
5	Families of Distributions
6	The Fourier Transform
7	The Fourier Transform for Tempered Distributions
8	First Midterm Exam
9	Weak, Distributional and Fundamental Solutions to Differential Equations
10	Review of Initial Value Problems for ODEs
11	Second-Order Boundary Value Problems for ODEs
12	Second-Order Boundary Value Problems for ODEs
13	The Adjoint Problem
14	Solvability Conditions
15	Modified Green Functions
16	Second Midterm Exam
17	Second Order PDEs and BVPs
18	Second Order PDEs and BVPs
19	Eigenfunction Expansions
20	Partial Eigenfunction Expansions
21	Partial Eigenfunction Expansions
22	The Method of Images
23	The Method of Images
24	The Boundary Element Method
25	Final Exam

Course Grade Components:

- First midterm exam: 25%
- Second midterm exam: 25%
- Final exam: 25%
- Course work: 25%

Honor Code Policy:

The rules for observing the Honor Code in this course are quite simple: you must never show any other student your written work. You are not allowed to write down formulas for another student, or to let them see your homework, or to demonstrate something to them on a blackboard or use any other type of written communication.

You are allowed to talk about the course work (the weekly assignments), but may not communicate in writing. For example, it is OK to tell another student “I solved this differential equation by substituting as for a homogeneous equation.” It is not OK to actually show another student the written calculations of how you did this.

Of course, during exams, no communication of any kind (verbal or written) is allowed!