Spring Term 2017

Vv557 Methods of Applied Mathematics II

Review Questions and Problems

JOINT INSTITUTE
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Class Session 10: Solvability Conditions and Modified Green's Functions

Video Files

32 Solvability Conditions.mp4
33 Modified Green Functions.mp4
34 Solution Formula via Modified Green Functions.mp4

Review Questions

i) State the Fredholm Alternative.

ii) State a solvability condition for a second-order inhomogeneous BVP for an ODE. What is the relationship to the adjoint problem?

iii) Explain what a modified Green Function is and what it is used for.

iv) Give the solution formula for a BVP using a modified Green function.

Exercises

Exercise 10.1. Find the solvability condition for

$$u^{(4)} = f, \quad 0 < x < 1, \qquad u(0) = u(1) = u''(0) = u''(1) = 0, \qquad u'(0) - u'(1) = \gamma.$$

Check your result by solving the differential equation with the four homogeneous conditions by using Green's function and then trying to satisfy the last boundary condition.

Exercise 10.2. Find the solvability condition for

$$-u'' - u = f, \quad -\pi < x < \pi, \qquad u(\pi) - u(-\pi) = \gamma_1, \qquad u'(\pi) - u'(-\pi) = \gamma_2.$$

Exercise 10.3.

i) Find the nontrivial solutions of

$$u^{(4)} = 0, \quad 0 < x < 1, \qquad u''(0) = u'''(0) = u''(1) = u'''(1) = 0$$

and give a physical interpretation in beam theory.

ii) Show that the problem is self-adjoint.

iii) Define and construct the modified Green's function.

iv) Solve $u^{(4)} = f$ with the homogeneous boundary conditions above when f satisfies the solvability conditions.

Exercise 10.4. Find the modified Green function for

$$L = \frac{d^2}{dx^2} + \pi^2, \quad 0 < x < 1, \qquad B_1 u = u(0) + u(1), \qquad B_2 u = u'(0) + u'(1)$$

Facultative Exercises

Exercise 10.5. Consider the boundary value operator given by

$$L = \frac{d^3}{dx^3}, \quad 0 < x < 1, \qquad B_1 u = u'(0), \qquad B_2 = u'(1), \qquad B_3 = u''(1)$$

i) Find $g(x, \xi)$.

ii) Express the solution of $Lu = f$, $B_k u = 0$, $k = 1, 2, 3$ in terms of Green's function.

iii) Find the adjoint boundary conditions.

Exercise 10.6. Consider the boundary value operator given by

$$L = \frac{d^4}{dx^4}, \quad 0 < x < 1, \qquad B_1 u = u(0), \qquad B_2 u = u'''(0), \qquad B_3 = u(1), \qquad B_4 = u'''(1)$$

What is the solvability condition for

$$u^{(4)} = f, \quad 0 < x < 1, \qquad B_1 u = B_2 u = B_3 u = B_4 u = 0?$$

Exercise 10.7. Find the solvability condition for

$$-u'' = f, \quad 0 < x < 1, \qquad u(1) - u(0) = \gamma_1, \qquad u'(1) = \gamma_2.$$

Does Green's function exist?