Spring Term 2017

Vv557 Methods of Applied Mathematics II

Review Questions and Problems

Class Session 4: The Fourier Transform

Video Files

15 The Fourier Transform and Functions of Rapid Decrease.mp4
16 Continuity of the Fourier Transform.mp4
17 The Fourier Inversion Formula and Properties of the Fourier Transform.mp4

Review Questions

i) How is the space of Schwartz functions defined?

ii) What is the relationship between the sets of test functions and Schwartz functions?

iii) Give three examples (not trivially equivalent) of Schwartz functions.

iv) How is the Fourier transform defined for Schwartz functions?

v) How is convergence defined in the space of Schwartz functions? What does "continuity of the Fourier transform" mean?

vi) List the basic properties of the Fourier transform and the convolution.

Exercises

Exercise 4.1. Calculate the Fourier transforms of the following elements in $L^1(\mathbb{R})$ (the theory of distributions is not needed):

i) $\Pi_{a,b}(x) = \begin{cases} 1 & a < x < b, \\ 0 & \text{otherwise}, \end{cases}$ $\qquad a, b \in \mathbb{R}.$

ii) $e^{-a|x|}$, $a > 0$.

iii) e^{-ax^2}, $a > 0$.

iv) $\cos(x)e^{-x^2}$.

v) $\cos(2x)/(4 + x^2)$.

vi) the convolution of xe^{-x^2} and e^{-x^2}.

Exercise 4.2. Suppose that $(f * g)(x) = 0$ for all x, where $f, g \in \mathcal{S}(\mathbb{R})$. Does this imply $f \equiv 0$ or $g \equiv 0$? What if $f = g$?