Spring Term 2017

Vv557 Methods of Applied Mathematics II

Review Questions and Problems

Class Session 8: Second-Order Boundary Value Problems for ODEs

Video Files

26 Second-Order Boundary Value Problems.mp4
27 Second-Order Boundary Value Problems with Separated Boundary Conditions.mp4
28 Green's Function and a Solution Formula for Second-Order Boundary Value Problems.mp4

Review Questions

i) What is a fully homogeneous BVP?

ii) What are mixed and umixed bopundary conditions?

iii) How is the Green function for a BVP for a second-order ODE defined?

iv) How is a solution to an umixed BVP constructed?

v) Explain how to find the functions u_1 and u_2 used to construct a solution for the general (mixed) BVP.

vi) Derive the solution formula for the general BVP. Explain how the function v is constructed to satisfy the boundary conditions.

Exercises

Exercise 8.1. Prove that the functions u_1 and u_2 used in the construction of the solution for an unmixed BVP are independent.

Exercise 8.2. Consider the boundary value operator given by

$$Lu = u'' + 2u' - 3u, \quad 0 < x < 1, \qquad B_1 u = u(0) + u'(0), \qquad B_2 u = u(1).$$

i) Find a fundamental solution for L on $(0, 1)$.

ii) Find a function u_1 satisfying $Lu_1 = 0$ and $B_1 u_1 = 0$.

iii) Find a function u_2 satisfying $Lu_2 = 0$ and $B_2 u_2 = 0$.

iv) Write down Green's function for (L, B_1, B_2).

v) Use the functions constructed above to write down the solution of

$$Lu = 1, \qquad B_1 u = 2, \qquad B_2 u = 1.$$

Exercise 8.3. Consider the boundary value operator given by

$$Lu = u'', \quad 0 < x < 1, \qquad B_1 u = u(0) - 2u(1), \qquad B_2 u = u'(0) - u(1).$$

i) Find a fundamental solution for L on $(0, 1)$.

ii) Find a function u_1 satisfying $Lu_1 = 0$ and $B_1 u_1 = 0$.

iii) Find a function u_2 satisfying $Lu_2 = 0$ and $B_2 u_2 = 0$.

iv) Write down Green's function for (L, B_1, B_2).