Spring Term 2017

Vv557 Methods of Applied Mathematics II

Review Questions and Problems

Class Session 9: Adjoint Boundary Value Problems and Higher Order Equations

Video Files

29 The Adjoint Second-Order Boundary Value Problem.mp4
30 The Adjoint Green Function for a Second-Order Problem.mp4
31 Boundary Value Problems of General Order.mp4

Review Questions

i) How are adjoint boundary conditions defined?

ii) What is the adjoint boundary value problem to a given BVP?

iii) What is the adjoint Green function?

iv) Explain the role of the conjunct in constructing the adjoint BVP.

v) Compare the solution formula using the conjunct with the solution formula obtained previously.

Exercises

Exercise 9.1. Consider the boundary value operator given by

$$L = \frac{d^2}{dx^2} + 4\frac{d}{dx} + 3, \quad a < x < b, \qquad B_1 u = u'(a) + 4u(a), \qquad B_2 u = u'(b) + 4u(b).$$

i) Find L^*.

ii) Find J.

iii) Find B_1^* and B_2^*.

Exercise 9.2. Consider the boundary value problem operator given by

$$L = \frac{d^2}{dx^2}, \quad 0 < x < 1, \qquad B_1 u = u(0).$$

Characterize M^* by three boundary functionals.

Exercise 9.3. Consider the boundary value operator given by

$$Lu = u'', \quad 0 < x < 1, \qquad B_1 u = u'(0) - u(1), \qquad B_2 u = u'(1).$$

i) Find $g(x, \xi)$

ii) It is obvious that $L = L^*$. Find the adjoint boundary conditions and calculate $g^*(x, \xi)$.

iii) Verify that $g^*(x, \xi) = g(\xi, x)$.

iv) Write down a solution formula using the conjunct for the problem (L, B_1, B_2) with data $(f; \gamma_1, \gamma_2)$.

Facultative Exercises

Exercise 9.4. Consider the boundary value operator given by

$$L = \frac{d^4}{dx^4}, \quad 0 < x < 1, \qquad B_1 u = u(0), \qquad B_2 u = u'''(0), \qquad B_3 = u(1), \qquad B_4 = u''(1)$$

i) Find $g(x, \xi)$.

ii) It is obvious that $L = L^*$. Find the adjoint boundary conditions and calculate $g^*(x, \xi)$.

iii) Show that $g(x, \xi) \neq g(\xi, x)$.

Exercise 9.5. Consider the boundary value problem given by

$$Lu := -u'' = f, \quad -1 < x < 1, \qquad B_1 u := \int_{-1}^{1} x u(x)\, dx = \gamma_1, \qquad B_2 u := \int_{-1}^{1} u(x)\, dx = \gamma_2$$

for f piecewise continuous on $[-1, 1]$ and $\gamma_1, \gamma_2 \in \mathbb{R}$. Find the corresponding Green's function and write down a solution formula for the problem.