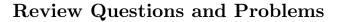
Vv557 Methods of Applied Mathematics II





# Class Session 9: Adjoint Boundary Value Problems and Higher Order Equations

## Video Files

29 The Adjoint Second-Order Boundary Value Problem.mp430 The Adjoint Green Function for a Second-Order Problem.mp431 Boundary Value Problems of General Order.mp4

### **Review Questions**

- i) How are adjoint boundary conditions defined?
- ii) What is the adjoint boundary value problem to a given BVP?
- iii) What is the adjoint Green function?
- iv) Explain the role of the conjunct in constructing the adjoint BVP.
- v) Compare the solution formula using the conjunct with the solution formula obtained previously.

#### Exercises

Exercise 9.1. Consider the boundary value operator given by

$$L = \frac{d^2}{dx^2} + 4\frac{d}{dx} + 3, \quad a < x < b, \qquad B_1 u = u'(a) + 4u(a), \qquad B_2 u = u'(b) + 4u(b).$$

- i) Find  $L^*$ .
- ii) Find J.
- iii) Find  $B_1^*$  and  $B_2^*$ .

Exercise 9.2. Consider the boundary value problem operator given by

$$L = \frac{d^2}{dx^2}, \quad 0 < x < 1, \qquad B_1 u = u(0).$$

Characterize  $M^*$  by three boundary functionals.

Exercise 9.3. Consider the boundary value operator given by

$$Lu = u'', \quad 0 < x < 1,$$
  $B_1 u = u'(0) - u(1),$   $B_2 u = u'(1).$ 

- i) Find  $g(x,\xi)$
- ii) It is obvious that  $L = L^*$ . Find the adjoint boundary conditions and calculate  $g^*(x,\xi)$ .
- iii) Verify that  $g^*(x,\xi) = g(\xi,x)$ .
- iv) Write down a solution formula using the conjunct for the problem  $(L, B_1, B_2)$  with data  $(f; \gamma_1, \gamma_2)$ .

#### **Facultative Exercises**

Exercise 9.4. Consider the boundary value operator given by

$$L = \frac{d^4}{dx^4}, \quad 0 < x < 1, \qquad B_1 u = u(0), \qquad B_2 u = u''(0), \qquad B_3 = u(1), \qquad B_4 = u''(1)$$

- i) Find  $g(x,\xi)$ .
- ii) It is obvious that  $L = L^*$ . Find the adjoint boundary conditions and calculate  $g^*(x,\xi)$ .
- iii) Show that  $g(x,\xi) \neq g(\xi,x)$ .

Exercise 9.5. Consider the boundary value problem given by

$$Lu := -u'' = f, \quad -1 < x < 1, \qquad B_1 u := \int_{-1}^1 x u(x) \, dx = \gamma_1, \qquad B_2 u := \int_{-1}^1 u(x) \, dx = \gamma_2$$

for f piecewise continuous on [-1, 1] and  $\gamma_1, \gamma_2 \in \mathbb{R}$ . Find the corresponding Green's function and write down a solution formula for the problem.