UNIVERSITY OF MICHIGAN

UM-SJTU JOINT INSTITUTE
上海交通大学交大密西根
联合学院

SHANGHAI JIAO TONG UNIVERSITY

Applied Calculus III

Exercise Set 1

Date Due: 12:00 PM, Thursday, the 20th of May 2010

Office hours: Wednesdays, 12:00-1:00 PM and on the SAKAI system

You are required to compose your solutions in *neat and legible handwriting.* Up to 10% of the total score may be deducted solely due to the apearance and legibility of your writing and your use of the English language.

In order to obtain the highest possible score, make sure that you explain your reasoning. Often, simple formulae are not enough to answer a question. *Explain what you are doing!* This will also ensure that you get a large fraction of the total points even if you make a mistake in your calculations. In short write simple, whole grammatical sentences that include a subject, verb and object.

Exercise 1.

i) Which complex numbers $z \in \mathbb{C}$ satisfy the inequality $|z+2| \le |z-1|$?

ii) Calculate z from the equality $(1+2i)z + (1-i)^2 = i - (2+i)z$.

iii) Sketch the following subsets of $\mathbb{C}$:

$$A = \{z \in \mathbb{C} \colon |z+1-i| + |z-1-i| = 6\}, \qquad B = \{z \in \mathbb{C} \colon |z+3| - |z-3| = 4\},$$
$$C = \{z \in \mathbb{C} \colon |z-1-i| = \operatorname{Re}(z+1)\}, \qquad D = \{z \in \mathbb{C} \colon |z-2-3i| = 4\}.$$

iv) Find all the roots of the equation
$$z^5 = 1 + 2i.$$

v) Prove the *parallelogram law* $|z_1+z_2|^2 + |z_1-z_2|^2 = 2(|z_1|^2 + |z_2|^2)$, $z_1, z_2 \in \mathbb{C}$.

(2 + 2 + 4 × 1 + 2 + 1 Marks)

Exercise 2. Use the Gauß-Jordan algorithm to find all solutions of the following systems of equations:

a)
$$\begin{array}{rcrcrcr} 4x_1 & + & 2x_2 & - & 2x_3 & = & -2 \\ -3x_1 & + & x_2 & & & = & 6 \\ x_1 & + & 4x_2 & + & 2x_3 & = & -9 \end{array}$$

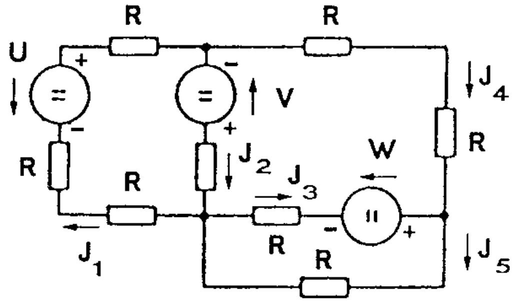
b)
$$\begin{array}{rcrcrcr} x_1 & - & 3x_2 & - & 5x_3 & = & 26 \\ 2x_1 & - & 2x_2 & + & x_3 & = & 12 \\ -3x_1 & + & 5x_2 & - & 6x_3 & = & 2 \end{array}$$

c)
$$\begin{array}{rcrcrcr} 3x_1 & + & x_2 & - & 2x_3 & = & 3 \\ 24x_1 & + & 10x_2 & - & 13x_3 & = & 25 \\ -6x_1 & - & 4x_2 & + & x_3 & = & -7 \end{array}$$

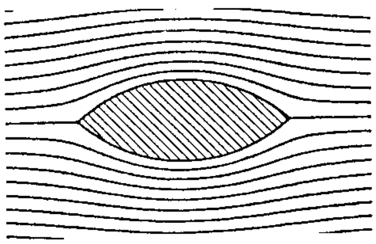
d)
$$\begin{array}{rcrcrcrcr} x_1 & + & 2x_2 & + & x_3 & + & x_4 & = & 0 \\ 2x_1 & + & x_2 & + & x_3 & + & 2x_4 & = & 0 \\ x_1 & + & 2x_2 & + & x_3 & + & x_4 & = & 0 \end{array}$$

(4 × 2 Marks)

Exercise 3. In the direct current (DC) circuit at right, $R = 330\,\Omega$, $U = V = 300\,\text{V}$ and $W = 200\,\text{V}$.
Use Kirchhoff's laws to find the currents J_1, J_2, J_3, J_4 and J_5!
(5 Marks)



Exercise 4. This exercise is concerned with the dimensional analysis of the flow resistance of a ship. We list the following physical quantities and their units:



- Density of water - $[\rho] = \mathrm{kg\,m^{-3}}$
- Speed of ship - $[v] = \mathrm{m\,s^{-1}}$
- Wetted surface - $[S] = \mathrm{m^2}$
- Mass of ship - $[m] = \mathrm{kg}$
- Brake acceleration - $[b] = \mathrm{m\,s^{-2}}$

i) Which formulas of the form $\rho^\alpha v^\beta S^\gamma m^\delta b^\varepsilon = C$ are possible if C is to be a dimensionless constant?

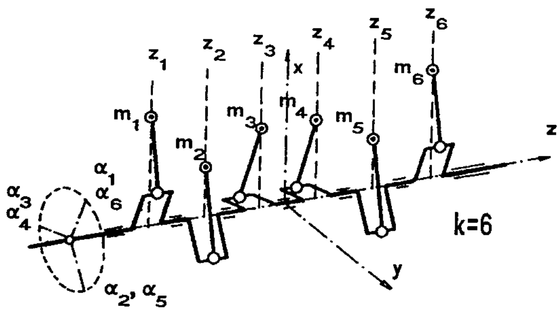
ii) Which formulas involving ρ, v, S are possible for the flow resistance $F = m \cdot b$?

(3 + 2 Marks)

Exercise 5. The 2nd order balancing for a k cylinder engine gives the following equations for the 1st and 2nd order momenta and moments of inertia:

$$\sum_{i=1}^k m_i \sin\alpha_i = 0, \quad \sum_{i=1}^k m_i \cos\alpha_i = 0, \quad \sum_{i=1}^k m_i \sin 2\alpha_i = 0, \quad \sum_{i=1}^k m_i \cos 2\alpha_i = 0,$$
$$\sum_{i=1}^k m_i z_i \sin\alpha_i = 0, \quad \sum_{i=1}^k m_i z_i \cos\alpha_i = 0, \quad \sum_{i=1}^k m_i z_i \sin 2\alpha_i = 0, \quad \sum_{i=1}^k m_i z_i \cos 2\alpha_i = 0,$$

Here $k > 1$ and for $i = 1, \ldots, k$ the α_i are the crank angles, $m_i > 0$ the masses, the center of mass is at $z = 0$ and $z_i \neq z_j$ whenever $i \neq j$.



i) Give the homogeneous linear system of equations that the vectors $m = (m_1, \ldots, m_k)$ and $(m_1 z_1, \ldots, m_k z_k)$ must satisfy.

ii) Consider the case of 4 cylinders ($k = 4$) with firing order 1–3–4–2, $\alpha_1 = \alpha_4 = 0$, $\alpha_2 = \alpha_3 = 180°$. Give and solve the homogeneous system of equations that results. Is a 2nd order balancing possible?

iii) Consider the case of 6 cylinders ($k = 6$) with firing order 1–5–3–6–2–4, $\alpha_1 = \alpha_6 = 0$, $\alpha_2 = \alpha_5 = 120°$, $\alpha_3 = \alpha_4 = 240°$. Give and solve the homogeneous system of equations that results. Is a 2nd order balancing possible?

iv) For a 4 cylinder engine with the usual configuration $m_1 = m_2 = m_3 = m_4 = m$ and $z_1 = -z_4 = 3z_2 = -3z_3$, do there exist crank angles $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ such that a 2nd order balancing is possible?

(2 + 3 + 3 + 3 Marks)

Exercise 6. Which of the following objects define subspaces of $\mathbb{R}^n$? Prove your assertions!

i) $(\{(x_1, \ldots, x_n) \in \mathbb{R}^n : x_1 \leq 0\}, +, \cdot)$,

ii) $(\{(x_1, \ldots, x_n) \in \mathbb{R}^n : x_1 x_n = 0\}, +, \cdot)$,

iii) $(\{(x_1, \ldots, x_n) \in \mathbb{R}^n : x_1 + 5x_2 = 0\}, +, \cdot)$,

(3 × 1 Mark)

Exercise 7. Prove that the set of functions

$$\{f : f(x) = a \sin(x + b)\}$$

is a subspace of the space of continuous functions, $C(\mathbb{R})$.
(2 Marks)