UNIVERSITY OF MICHIGAN

UM-SJTU JOINT INSTITUTE
上海交通大学交大密西根
联合学院

SHANGHAI JIAO TONG UNIVERSITY

Applied Calculus II

Exercise Set 1

Date Due: 10:00 AM, Saturday, the 25th of September 2008

Office hours: Tuesdays and Thursdays, 12:00-2:00 PM and on the SAKAI system

A word of introduction

This is the first Exercise Set for your first Calculus course at university. You will find the style of the exercises and the requirements of the solutions quite different from what you are used to from school. Some of the exercises on this set (and all further sets) will be easy, some will be quite difficult. As with many problems in real life, you are not told in advance which are simple and which are more involved, but are supposed to find out for yourself. You will find it easier to obtain full marks for the exercises if you follow a few simple principles:

i) You are required to *hand in the exercises on time.* No exercises will be accepted after the due date.

ii) You are required to compose your solutions in *neat and legible handwriting.* 10% of the total score will be awarded solely for the apearance and legibility of your writing and your use of the English language (see next point).

iii) In order to obtain the highest possible score, make sure that you explain your reasoning. Often, simple formulae are not enough to answer a question. *Explain what you are doing!* This will also ensure that you get a large fraction of the total points even if you make a mistake in your calculations. In short write simple, whole grammatical sentences that include a subject, verb and object.

iv) You are **forbidden to use the symbols ∴ and ∵** in your writing. Write English sentences instead. You will **lose all points** for an exercise that includes these two symbols, no matter whether the solution is correct or not! On this subject, I would like to quote the great mathematician Serge Lang:

> It seems to me essential that students be required to write their mathematics papers in full and coherent sentences. A large portion of their difficulties with mathematics stems from their slapping down mathematical symbols and formulas isolated from a meaningful sentence and appropriate quantifiers. Papers should also be required to be neat and legible. They should not look as if a stoned fly had just crawled out of an inkwell. Insisting on reasonable standards of expression will result in drastic improvements of mathematical performance.
>
> S. Lang, *A First Course in Calculus*

v) You are encouraged to cooperate with other students. Feel free to discuss problems and develop solutions in groups. But *you are not allowed to simply copy other students' work!* The golden rule is: Feel free to discuss problems orally, but do not look at the written work of another student. The Teaching Assistants will report any suspected instances of this to me as a potential violation of the JI's Honor Code.

vi) If you have any problems, questions or comments regarding the exercises, the lecture or mathematics, please *visit me during my office hours.* This time has been set aside specifically for you, and you should make as much use fo it as you can. If you wish for comments to reach me anonymously, please talk to the Teaching Assistants.

With these words, I now let you commence your first exercises. Good luck!

Horst Hohberger

Exercise 1. Make a two-column table: in the left column, write out the letters of the Greek alphabet (lowercase and uppercase) in the correct order and in the right column write out the English names for the letters. It should start like this:

Greek letter	English name
α, A	alpha
β, B	beta
γ, $\varGamma$	gamma
$\vdots$	$\vdots$

(2 Marks)

Exercise 2. Prove the following statements by induction:

i) Let $n \in \mathbb{N}$, $n \geq 1$. Show that $\sum_{j=1}^{n} j^3 = \left(\sum_{j=1}^{n} j\right)^2$

ii) Let $n \in \mathbb{N}$. Show that $\prod_{j=1}^{n-1} \left(1 + \frac{1}{j}\right)^j = \frac{n^n}{n!}, \qquad n \geq 2.$

(2 + 2 Marks)

Exercise 3. For each of the following inequalities, find the sets of all $x \in \mathbb{R}$ satisfying the inequality.

i) $|x+2| \leq |x-1|$ ii) $|2 - |x+1|| \leq 1$ iii) $x^3 + 2x^2 - x - 2 > 0$

(3 × 2 Marks)

Exercise 4. Let $a, b, c, d \in \mathbb{R}$. Show that

i) $\frac{a}{b} < \frac{c}{d}$ and $b > 0$, $d > 0$ implies $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$;

ii) $a > 0$ and $b > 0$ implies $\sqrt{ab} \leq \frac{a+b}{2}$;

iii) $a > 0$ and $b > 0$ implies $\frac{a}{b} + \frac{b}{a} \geq 2$.

(1 + 1 + 1 Marks)

Exercise 5.

i) Let $u = 2 + 3i$, $v = 5 - i$ and $w = 1 + i$. Calculate $|u|$, $u + v$, $u - v$, $u \cdot v$, $\frac{u}{v}$ and $u^2 + 2vw$.

ii) For complex numbers $z_1, z_2 \in \mathbb{C}$, prove that $|z_1 z_2| = |z_1||z_2|$.

iii) Which complex numbers $z \in \mathbb{C}$ satisfy the inequality $|z+2| \leq |z-1|$?

iv) Calculate z from the equality $(1+2i)z^2 + (1-i)^2 = i - (2+i)z$.

v) Sketch the following subsets of $\mathbb{C}$:

$$A = \{z \in \mathbb{C} \colon |z+1-i| + |z-1-i| = 6\}, \qquad B = \{z \in \mathbb{C} \colon |z+3| - |z-3| = 4\},$$
$$C = \{z \in \mathbb{C} \colon |z-1-i| = \mathrm{Re}(z+1)\}, \qquad D = \{z \in \mathbb{C} \colon |z-2-3i| = 4\}.$$

vi) Prove the *parallelogram law* $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$, $z_1, z_2 \in \mathbb{C}$.

($6 \times \frac{1}{2} + 1 + 2 + 2 + 4 \times 1 + 2$ Marks)

Exercise 6. Perform the following polynomial division:

$$(3x^4 + 7x^3 + x^2 + 5x + 1) \div (x^2 + 1)$$

(2 Marks)

Exercise 7.
For each of the following functions f_k, $k = 1, \dots, 3$ find the translated functions $T_3 f_k$ and the dilated functions $D_2 f_k$. Give the domain and range of $f_k, T_3 f_k$ and $D_2 f_k$ and sketch each of $f_k, T_3 f_k$ and $D_2 f_k$.

$$f_1(x) = \frac{3 + |x|}{x}, \qquad f_2(x) = \frac{4 - x^2}{2 - x}, \qquad f_3(x) = \begin{cases} -1 & x \leq -1, \\ 3x + 2 & -1 < x < 1, \\ 7 - 2x & x \geq 1 \end{cases}$$

(3 × 3 Marks)