University of Michigan

交大密西根学院
UM–SJTU Joint Institute

Shanghai Jiao Tong University

Applied Calculus II

Exercise Set 10

Date Due: 4:00 PM, Tuesday, the 14th of December 2010

Office hours: Tuesdays and Thursdays, 12:00-2:00 PM and on the SAKAI system

Exercise 1. Find the length of the curves given by the following equations:

i) $y^2 = 4(x+4)^3, \quad 0 \le x \le 2, \ y > 0,$

ii) $y = \dfrac{x^2}{2} - \dfrac{\ln x}{4}, \quad 2 \le x \le 4,$

iii) $y = \ln(\cos x), \quad 0 \le x \le \pi/3,$

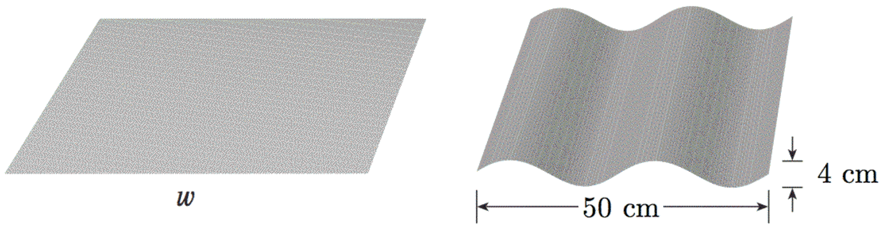
iv) $y = \ln x, \quad 1 \le x \le \sqrt{3}, \ y > 0,$

v) $y^2 = 4x, \quad 0 \le x \le 2,$

vi) $y = \ln\left(\dfrac{e^x+1}{e^x-1}\right), \quad 0 < a \le x \le b.$

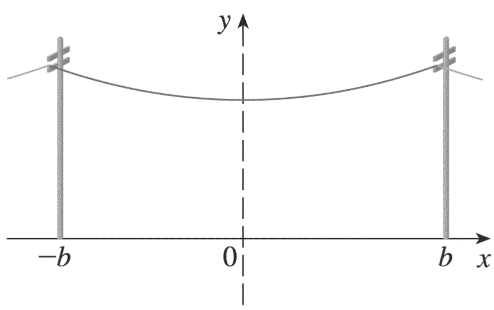
(6 × 2 Marks)

Exercise 2. A manufacturer of corrugated metal roofing wants to produce panels that are 50 cm wide and 4 cm thick by processing flat sheets of metal as shown below.



The profile of the roofing takes the shape of a sine wave. Find the width w of a flat metal sheet that is needed to make a 50-cm panel. You may use computer software to help solve this problem.
(3 Marks)

Exercise 3. The figure shows a telephone wire hanging between two poles at $x = -b$ and $x = b$. It takes the shape of a catenary with equation $y = c + a\cosh(x/a)$.



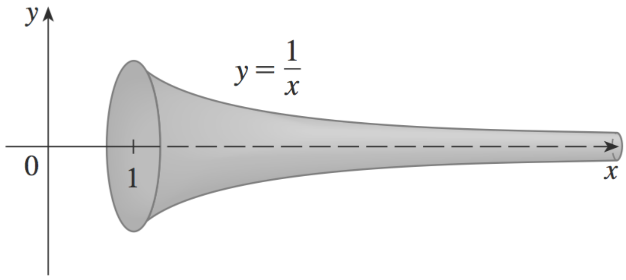
i) Find the length of the wire.

ii) Suppose two telephone poles are 50 ft apart and the length of the wire between the poles is 51 ft. If the lowest point of the wire must be 20 ft above the ground, how high up on each pole should the wire be attached? (You may use computer software to help solve this problem..)

(3 + 3 Marks)

Exercise 4.

i) Show that the surface area of *Gabriel's Horn*, shown below, is infinite. (Note that the enclosed volume is finite.)

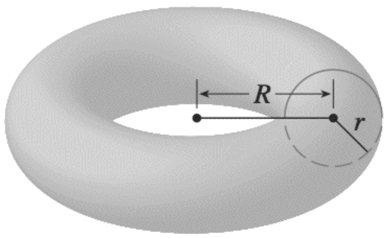


ii) If the infinite curve $y = e^{-x}$, $x \geq 0$, is rotated about the x-axis, find the area of the resulting surface.

(2 + 2 Marks)

Exercise 5. A group of engineers is building a parabolic satellite dish whose shape will be formed by rotating the curve $y = ax^2$, $a > 0$, about the y-axis. If the dish is to have a diameter of 3 m and a maximum depth of 60 cm, find the value of a and the surface area of the dish.
(3 Marks)

Exercise 6. Find the surface area of the torus pictured below:



(2 Marks)

Exercise 7. Show that the surface area of a zone of a sphere that lies between two parallel planes is $S = \pi dh$, where d is the diameter of the sphere and h is the distance between the planes. (Notice that S depends only on the distance between the planes and not on their location, provided that both planes intersect the sphere.)
(4 Marks)

Exercise 8. The *Folium of Descartes* is the set of points $\mathcal{F} = \{(x, y) \in \mathbb{R}^2, (x, y) = \gamma(t),\ t \in \mathbb{R}\}$, where

$$\gamma(t) = \left(\frac{3t}{1+t^3}, \frac{3t^2}{1+t^3} \right).$$

i) Show that if $(a, b) \in \mathcal{F}$, then $(b, a) \in \mathcal{F}$, i.e., $\mathcal{F}$ is symmetric with respect to the line $\{(x, y) \in \mathbb{R}^2 \colon x = y\}$. Where does $\mathcal{F}$ intersect this line?

ii) Find the points on $\mathcal{F}$ where the tangents are horizontal or vertical.

iii) Show that the line $\{(x, y) \in \mathbb{R}^2 \colon y = -x - 1\}$ is a slant asymptote.

iv) Sketch $\mathcal{F}$.

v) Show that $\mathcal{F} = \{(x, y) \in \mathbb{R}^2 \colon x^3 + y^3 = 3xy\}$.

vi) Find the area enclosed by the loop of $\mathcal{F}$.

(1 + 1 + 1 + 1 + 2 Marks)