Applied Calculus II

Exercise Set 4

Date Due: 10:00 AM, Thursday, the 28th of October 2010

Office hours: Tuesdays and Thursdays, 12:00-2:00 PM and on the SAKAI system

Exercise 1. The actual definition for the statement $f(x) = O(g(x)$ as $x \to x_0$ is the following: there exist constants $C > 0$ and $\varepsilon > 0$ such that

$$f(x) < C \cdot g(x) \qquad \text{for all } x \text{ with } |x - x_0| < \varepsilon.$$

(It can be shown that if $\lim_{x \to x_0} f(x)/g(x) = \alpha$ for some $\alpha \in \mathbb{R}$, then the above statement is true. However, the converse is not necessarily the case.) Similarly, the actual definition for the statement $f(x) = o(g(x)$ as $x \to x_0$ is the following: for every $C > 0$ there exists an $\varepsilon > 0$ such that

$$f(x) \le C \cdot g(x) \qquad \text{for all } x \text{ with } |x - x_0| < \varepsilon.$$

(This statement is actually equivalent to $\lim_{x \to x_0} f(x)/g(x) = 0$.)

Interpret and prove the following relations as $x \to x_0 \in \mathbb{R}$:

$$\begin{aligned} O(f(x)) + O(g(x)) &= O(|f(x)| + |g(x)|), \\ O(f(x))O(g(x)) &= O(f(x)g(x)), \\ O(f(x))o(g(x)) &= o(f(x)g(x)), \\ O(O(f(x))) &= O(f(x)), \\ o(O(f(x)) &= o(f(x)). \end{aligned}$$

(1 + 1 + 1 + 2 + 2 Marks)

Exercise 2. Let $f\colon \mathbb{R} \to \mathbb{R}$ be a real function.

i) Prove that if f is even, then f' is odd.

ii) Prove that if f is odd, then f' is even.

(1 + 1 Marks)

Exercise 3. Differentiate the following functions $f\colon \Omega \to \mathbb{R}$, where Ω is their maximal domain:

$$f(x) = \sqrt{30}, \qquad f(x) = -4x^{10}, \qquad f(x) = 5x^8 - 2x^5 + 6, \qquad f(x) = \sqrt{x} - \frac{1}{\sqrt{x}},$$

$$f(x) = \sqrt{x}(1 - x), \qquad f(x) = \frac{x^2 - 2\sqrt{x}}{x}, \qquad f(x) = \sqrt{3}x - \sqrt{2x}, \qquad f(x) = \sqrt[3]{x^2} - \sqrt[2]{x^3}$$

$$f(x) = (x^2 + 1)\sqrt[3]{x^2 + 2}, \qquad f(x) = \sqrt{x + \sqrt{x + \sqrt{x}}}, \qquad f(x) = \frac{(x-1)^4}{(x^2 + 2x)^5}, \qquad f(x) = \frac{1}{\sqrt{1 + x^2}}$$

(12 × 1 Mark)