Vv556 Methods of Applied Mathematics I

Linear Operators

Assignment 11

Date Due: 2:00 PM, Tuesday, the 12th of December 2017

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This assignment has a total of **(20 Marks)**.

Exercise 11.1
It is known that the operator

$$T\colon L^2([0,1]) \to L^2([0,1]), \qquad (Tu)(x) = \int_0^x u(y)\,dy$$

is compact but has no eigenvalues.

i) Explain why for any compact operator T, the oeprator T^*T is self-adjoint, compact and positive.
(3 Marks)

ii) Find T^* and verify that T^* is also compact.
(2 Marks)

iii) Find the eigenfunctions and eigenvalues of T^*T. (You may, for example, translate an integral equation into a boundary value problem).
(4 Marks)

iv) Find the singular value decomposition of T.
(3 Marks)

Exercise 11.2
Let $A \in \mathrm{Mat}(n \times n; \mathbb{C})$. Show how the polar decomposition of A can be obtained from the singular value decomposition.
(2 Marks)

Exercise 11.3
Give the compact singular value decomposition of

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \end{pmatrix}.$$

(3 Marks)

Exercise 11.4
Give the polar decomposition of

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}.$$

(3 Marks)