Course Description

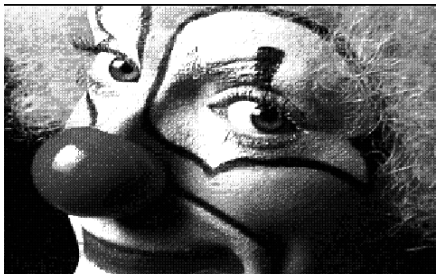
Vv556 Methods of Applied Mathematics I

Linear Operator Theory

Prerequisites: Graduate standing or permission of instructor.

Course website: http://umji.sjtu.edu.cn/personal/horst/teaching/vv556.html

Intended Audience: ME and ECE undergraduate and graduate students. Knowledge of elementary linear algebra and calculus is assumed. The course is accessible to junior students; no additional background is required.



Picture of a clown

Clown picture compressed using singular value decomposition

Description: This course gives an introduction to the theory of bounded linear maps on finite- and infinite-dimensional spaces.

In the first part, notions of linear algebra are reviewed and extended to infinite-dimensional vector spaces. This includes concepts such as scalar products, norms and (Schauder-) bases. As an application, Legendre polynomials, introduced as an orthonormalization of the monomials on the interval $[-1, 1]$ are introduced, and their role in multipole expansions is explored. Next, Hilbert spaces are introduced, leading to spaces of square-integrable functions and Fourier series. A look back and comaprison of the obtained results with the finite-dimensional cases of linear algebra concludes this part.
The second part focuses on bounded linear maps on (infinite-dimensional) spaces, introducing the matrix elements of such operators and using these to define Hilbert-Schmidt operators for square-summable sequences and square-integrable functions. The notions of inverses and adjoints of bounded linear operators are discussed and the spectrum of such operators is introduced. Compact operators are introduced and, motivated by a question from the theory of partial differential equations, the spectral theorem for compact operators is established.
The last part is dedicated to applications of the spectral theory, including the Rayleigh-Ritz method (applied specifically to Sturm-Liouville eigenvalue problems) and the polar and singular value decompositions of compact operators, which of course includes these decompositions for matrices.

Keywords: Metric spaces, open sets, dense sets and separable metric spaces, sequences in metric spaces and completeness, vector spaces, Banach spaces, Hilbert spaces, Fourier series, Legendre polynomials, multipole expansion, matrices, eigenvalue problem, matrix decompositions and applications, basic properties of bounded linear operators on Hilbert spaces, the spectrum, Rayleigh-Ritz method, spectral theorem for compact operators, Sturm-Liouville boundary value problems

Textbooks:

[J] K. Jänich, *Linear Algebra*, Springer 1994

[S] I. Stakgold and M. Holst, *Green's Functions and Boundary Value Problems*, 3rd Ed., Wiley 2011

[K] E. Kreyszig, *Introductory functional Analysis*, Wiley 1989

Syllabus:

Lecture	Lecture Subject
1	Introduction
2	Normed Vector Spaces
3	Bases and Inner Product Spaces
4	Bases and Inner Product Spaces
5	Legendre Polynomials and Applications
6	Legendre Polynomials and Applications
7	Hilbert Spaces
8	The Space of Square-Integrable Functions
9	Fourier Series
10	Finite-Dimensional Vector Spaces
11	**First Midterm Exam**
12	Linear Functionals and Operators
13	Matrix Elements and Hilbert-Schmidt Operators
14	Inverse and Adjoint of Bounded Linear Operators
15	The Spectrum
16	The Spectrum
17	Compact Operators
18	Spectral Theorem for Compact Operators
19	Spectral Theorem for Compact Operators
20	**Second Midterm Exam**
21	Sturm-Liouville Boundary Value Problems
22	The Rayleigh-Ritz Method
23	Positive Operators and the Polar Decomposition
24	The Singular Value Decomposition for Compact Operators and Matrices
25	**Final Exam**

Course Grade Components:

- First midterm exam: 30%
- Second midterm exam: 30%
- Final exam: 40%

Honor Code Policy:

Students within each group may discuss and cooperate freely when solving homework problems. It may happen that you find the solution of a homwork problem in some outside source (book, internet site, etc.). In that case *you are not allowed to just copy the solution*; this is considered a violation of the Honor Code.

The correct way of using outside sources is to understand the contents of your source and then to write in your own words and without referring back to the source the solution of the problem. Your solution should differ in style significantly from the published solution. *If in doubt, cite the source that you used.*

It is acceptable to discuss homework problems orally with members of other groups, but you may in no case look at each others' written notes. *Do not show your written solutions to any members of other groups.* This is considered a violation of the Honor Code.

Coursework and Assignment Groups

There will be weekly coursework (assignments) throughout the term, except in the first week. Students will be randomly assigned into *assignment groups* of three students and are expected to collaborate within each group and hand in a single, common solution paper to each coursework.

Each group must achieve *60%* of the total coursework points by the end of the term in order to obtain a passing grade for the course. However, the assignment points have *no effect on the course grade.*

Each member of an assignment group will receive the same number of points for each submission. However, there will be an opportunity for team members to anonymously evaluate each others' contributions to the assignments. In cases where one or more group members consistently do not contribute a commensurate share of the work, a TA or the instructor will investigate the situation and individual group members may lose some or all of their marks.

Assignments must be handed in on time, by the date given on each set of course work. At the sole discretion of the TAs, late homework may be accepted on the same day after the due time. Afetr that, all late homework must be submitted to the instructor personally with an explanation for the lateness. Late assignments will be accepted at the discretion of the instructor.

Up to 10% of the awarded marks for an assignment may be deducted for messiness or sloppy handwriting. Assignments typeset in LaTeXwill receive a 10% bonus on the awarded marks.

Students within an assignemnt group are obliged to collaborate with each other. Each assignment is handed in in the name of all group members, and all group members are jointly responsible for the submitted work. Sanctions for Honor Code violations will in general apply to all group members equally.