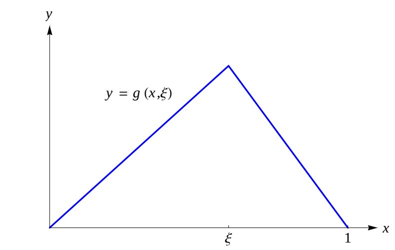
Course Description

Vv557 Methods of Applied Mathematics II
Advanced Methods for PDEs

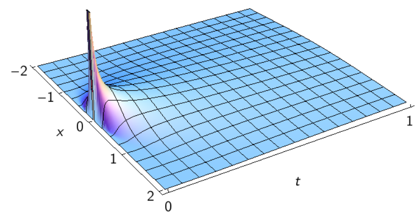
Spring Term 2017

Prerequisites: Vv256 or Vv286 and the preceding calculus courses or graduate standing or permission of instructor.

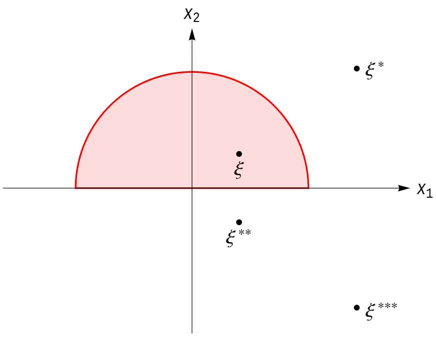
Intended Audience: ME and ECE undergraduate and graduate students. The course is accessible to sophomore students; no additional background is required. Basic knowledge of PDEs (e.g., as contained in *Vv454 Partial Differential Equations and Boundary Value Problems*) is very helpful.



Non-Smooth Solutions to Differential Equations



Delta Families and Point Sources



Green functions via Method of Images

Description: The course consists of three parts, each dealing with certain mathematical techniques useful for solving differential equations. Examples from mechanical as well as electrical engineering will be used throughout.

Our initial motivation is the desire to understand the treatment of point sources. Starting from the Dirac delta function as a formal symbol to denote a point source, we begin a formal treatment of generalized functions (distributions), including principal value integrals, notions of convergence and delta families, the distributional Fourier transform and solutions of distributional equations.

The second part of the course applies the theory of distributions to ordinary differential equations (ODEs). Strong, weak and distributional solutions are introduced and general solution formulas obtained. The main focus is then on obtaining Green's functions for boundary value problems (BVPs) for ODEs, leading to a brief discussion of solvability and modified Green's functions for ODEs.

The final third of the course extends the ODE methods to PDEs. Green's formulas for boundary value problems of the first, second and third kind are derived. Subsequently, methods for finding Green's functions are explored, including that of full and partial eigenfunction expansions, the method of images and (if time permits) conformal mappings. Finally, a short introduction to the use of Green's functions for the Laplace equation in the boundary element method (BEM) is presented.

Keywords: Point sources, distributions, classical, weak and fundamental solutions to differential equations, causal fundamental solutions, formal adjoint differential operator, conjunct, Green's formula, Green functions for boundary value problems for ODEs, modified Green functions for ODEs, solution formulas for ODEs and PDEs, Green functions for PDEs, eigenfunction expansions, method of images, conformal mappings, boundary element method.

Textbooks:

- Y. Pinchover and J. Rubinstein, *An Introduction to Partial Differential Equations*, Cambridge University Press 2005,
- I. Stakgold and M. Holst, *Green's Functions and Boundary Value Problems*, 3^{rd} Ed., Wiley 2011,
- E. Zauderer, *Partial Differential Equations of Applied Mathematics*, 2^{nd} Ed., Wiley 1989,

Syllabus:

Lecture	Lecture Subject
1	Introduction
2	Point Sources and Green Functions
3	Distributions
4	Operations on Distributions
5	Families of Distributions
6	The Fourier Transform
7	The Fourier Transform for Tempered Distributions
8	**First Midterm Exam**
9	Weak, Distributional and Fundamental Solutions to Differential Equations
10	Review of Initial Value Problems for ODEs
11	Second-Order Boundary Value Problems for ODEs
12	Second-Order Boundary Value Problems for ODEs
13	The Adjoint Problem
14	Solvability Conditions
15	Modified Green Functions
16	**Second Midterm Exam**
17	Second Order PDEs and BVPs
18	Second Order PDEs and BVPs
19	Eigenfunction Expansions
20	Partial Eigenfunction Expansions
21	Partial Eigenfunction Expansions
22	The Method of Images
23	The Method of Images
24	The Boundary Element Method
25	**Final Exam**

Course Grade Components:

- First midterm exam: 25%
- Second midterm exam: 25%
- Final exam: 25%
- Course work: 25%

Starting from the third class, this course uses the "flipped classroom" model of teaching. Class time will feature quizzes, in-class assignments, presentations and other activities.

You will be asked to read through course slides or watch lecture videos in your own time and to prepare certain questions and/or assignments for presentation in class.

The course work component of the course grade will be based primarily on the work done during the class periods. You will form groups of 4-5 students. Members of each group may cooperate freely when preparing for class or doing homework.

The class work grade will be shared among all members of a group (excluding scores of quizzes and other individually assessed work). In particular, the performance of a single group member in class will reflect on all other group members.

Honor Code Policy:

Students within each group may discuss and cooperate freely when solving homework problems. It may happen that you find the solution of a homwork problem in some outside source (book, internet site, etc.). In that case *you are not allowed to just copy the solution*; this is considered a violation of the Honor Code.

The correct way of using outside sources is to understand the contents of your source and then to write in your own words and without referring back to the source the solution of the problem. Your solution should differ in style significantly from the published solution. *If in doubt, cite the source that you used.*

It is acceptable to discuss homework problems orally with members of other groups, but you may in no case look at each others' written notes. *Do not show your written solutions to any members of other groups.* This is considered a violation of the Honor Code.