Discrete Mathematics

Assignment 4

Date Due: 8:00 PM, Thursday, the 16th of June 2011

Office hours: Tuesdays, 1:00-3:00 PM, and Wednesdays, 12:00-1:00 PM

Exercise 1. Order the letters M,I,C,H,I,G,A,N alphabetically using

i) merge sort,

ii) insertion sort,

iii) bubble sort

algorithms. (Note that it does not matter that the letter "I" is repeated.) For each algorithm, show what the arrangement is after each pass/merge. How many comparisons are made using each algorithm?
(3 × 3 Marks)

Exercise 2. Verify that the program segment

if $x < y$ **then** $min := x$
else $min := y$

is correct with respect to the initial assertion $p\colon T$ and the final assertion $q\colon (x < y \wedge min = x) \vee (x > y \wedge min = y)$. N.B.: you have to show that the program is partially correct as well as that it actually terminates.
(3 Marks)

Exercise 3. Use a loop invariant to prove that the following program segment for comupting the nth power, $n \in \mathbb{Z}_+$, of $x \in \mathbb{R}$ is correct:

$power := 1$
$i := 1$
while $i \leq n$ **do**
  loop
    $power := power * x$
    $i := i + 1$
  end loop
end while

N.B.: you have to show that the program is partially correct as well as that it actually terminates.
(4 Marks)

Exercise 4. This program computes quotients and remainders

$r := a$
$q := 0$
while $r \geq d$ **do**
  loop
    $r := r - d$
    $q := q + 1$
  end loop
end while

Verify that it is partially correct with respect to the initial assertion $P\colon a, d \in \mathbb{Z}_+$ and the final assertion $Q\colon (r = a \bmod d) \wedge (q = a \div b)$.
(4 Marks)

Exercise 5. Prove that the product of any three consecutive integers is divisible by 6.
(3 Marks)

Exercise 6. The *iterated integer sum* of $n \in \mathbb{Z}_+$ is calculated as follows: The decimal digits of n are added to yield a sum n_1. If n_1 is greater than 9, the integers of n_1 are added. This process is repeated until a number between 0 and 9 is obtained. For example, the iterated integer sum of 54469 is calculated as follows: $5+4+4+6+9=28$, $2+8=10$, $1+0=1$.

i) Write pseudocode for an iterative algorithm to find the iterated integer sum.

ii) Give the worst-case number of additions that the program needs to perform to calculate the iterated integer sum of $n \in \mathbb{Z}_+$.

iii) Prove or disprove the following: the iterated integer sum of a product of two numbers is equal to the iterated integer sum of the product of their iterated integer sums.

(2+2+3 Marks)