Ve401 Probabilistic Methods in Engineering

Summer 2017 — Assigment 1

Date Due: 12:10 PM, Wednesday, the 24$^{\text{th}}$ of May 2017

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This assignment has a total of **(32 Marks)**.

Exercise 1.1 Sample Space, Events, Probabilities

To get the opportunity to enter the McNeill River Brown (Grizzly) Bear Sanctuary in Alaska, one must enter a lottery. For a given year there are 2000 individuals entered, and fof these a set of 120 names will be randomly selected. Assume that you and a friend are both entered into the lottery.

Write down a suitable sample space for the selection results of yourself and your friend.

i) Write down a suitable sample space for the selection results of yourself and your friend. Characterize the events that

- both of you are chosen
- only one of you is chosen
- neither of you is chosen

as events of the sample space.
(2 Marks)

ii) Find the probabilities for the three events above.
(3 Marks)

iii) If your friend is chosen, what is the probability that you are also chosen?
(2 Marks)

Exercise 1.2 DAlembert's Approach to Coin Tossing

The French mathematician Jean DAlembert claimed that in tossing a coin twice (or two coins at once), we have only three possible outcomes: "two heads," "one head," and "no heads." This is a legitimate sample space, of course. However, DAlembert also claimed that each outcome in this space has the same probability 1/3.

i) Is it possible to have a coin biased in such a way so as to make DAlemberts claim true? If so, how? If not, why not?
(2 Marks)

ii) Is it possible to make two coins with different probabilities of heads so as to make DAlemberts claim true if both coins are tossed at once? If so, how? If not, why not?
(2 Marks)

Exercise 1.3

The ablity to observe and recall details is important in science. Unfortunately, the power of suggestion can distort memory. A study of recall is conducted as follows: Subjects are shown a film in which a car is moving along a country road. There is no barn in the film. The subjects are then asked a series of questions concerning the film. Half the subjects are asked, "How fast was the car moving when it passed the barn?" The other half is not asked the question. Later each subject is asked, "Is there a barn in the film?" Of those asked the first question concerning the barn, 17% answer "yes"; only 3% of the others answer "yes."

i) What is the probability that a randomly selected participant in this study claims to have seen the non-existent barn?
(2 Marks)

ii) Is claiming to see the barn independent of being asked the first question about the barn?
(2 Marks)

Exercise 1.4 "Almost Sure" Occurrences of Events

A coin is tossed preatedly until it turns up "heads". The number of tosses required for this first appearance of heads is recorded (so the sample space is $\mathbb{N} \setminus \{0\}$).

i) Find the probability that the coin turns up "heads" on the nth toss, $n \in \mathbb{N} \setminus \{0\}$.
(2 Marks)

ii) What is the probability that the coin turns up "heads" eventually?
(2 Marks)

iii) Comment on the meaning of the phrase "almost surely". Is an event that occurs with probability $P = 1$ actually *certain* to happen? [Write at least two full, grammatical English sentences.]
(2 Marks)

Exercise 1.5 Probabilities of Events

Let A and B be events in a sample space S. Use the axioms of probability to show the following statements:

i) Let $A \subset B$. Show that $P[A] \leq P[B]$.
(1 Mark)

ii) Assume that A and B are independent and that $P[A]P[B] > 0$. Show that A and B are not mutually exclusive.
(2 Marks)

iii) Show that $P[A \cup B] = P[A] + P[B] - P[A \cap B]$
(1 Mark)

Exercise 1.6 Bayes's Theorem and a Subtle Problem Phrasing

It is reported that 50% of all computer chips produced are defective. Inspection assures that only 5% of the chips legally marketed are defective. Unfortunately, some chips are stolen before inspection. If 1% of all chips on the market are stolen, find the probability that a given chip is stolen given that it is defective.
(4 Marks)

Exercise 1.7 Another Perspective on the Monty Hall Paradox

One[1] of three prisoners, A, B, and C, is to be executed the next morning. They all know about it, but they do not know who is going to die. The warden knows, but he is not allowed to tell them until just before the execution.

In the evening, one of the prisoners, say A, goes to the warden and asks him: "Please, tell me the name of one of the two prisoners, B and C, who is not going to die. If both are not to die, tell me one of their names at random. Since I know anyway that one of them is not going to die, you will not be giving me any information."

The warden thought about it for a while, and replied: "I cannot tell you who is not going to die. The reason is that now you think you have only 1/3 chance of dying. Suppose I told you that B is not to be executed. You would then think that you have a 1/2 chance of dying, so, in effect, I would have given you some information."

Was the warden right or was the prisoner right?
(3 Marks)

[1] Exercise 4.4.9 of R. Bartoszynski and M. Niewiadomska-Buga *Probability and Statistical Inference*, Second Edition, Wiley 2008