

Ve401 Probabilistic Methods in Engineering

Summer 2017 — Assignment 3

Date Due: 12:10 PM, Wednesday, the 7th of June 2017



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This assignment has a total of (22 Marks).

Exercise 3.1 Continuous Uniform Distribution

A continuous random variable X is said to be *uniformly distributed* over an interval (a, b) if its density is given by

$$f(x) = \begin{cases} 1/(b-a) & \text{for } a < x < b, \\ 0 & \text{otherwise.} \end{cases}$$

- i) Show that this is a density for a continuous random variable.
(1 Mark)
- ii) Sketch the graph of the density and shade the area of the graph that represents $P[X \leq (a+b)/2]$.
(1 Mark)
- iii) Find the probability pictured in part ii).
(1 Mark)
- iv) Let (c, d) and (e, f) be subintervals of (a, b) of equal length. What is the relationship between $P[c \leq X \leq d]$ and $P[e \leq X \leq f]$?
(1 Mark)
- v) Find the cumulative distribution function F for a uniformly distributed random variable.
(1 Mark)
- vi) Show that $E[X] = (a+b)/2$ and $\text{Var } X = (b-a)^2/12$.
(2 Marks)

Exercise 3.2 Finding Probabilities with the Normal Distribution

The compressive strength of samples of cement can be modeled by a normal distribution with a mean of 6000 kilograms per square centimeter and a standard deviation of 100 kilograms per square centimeter.

- i) What is the probability that a samples strength is less than 6250 kg / cm²?
(1 Mark)
- ii) What is the probability that a samples strength is between 5800 and 5900 kg / cm²?
(1 Mark)
- iii) What strength is exceeded by 95% of the samples?
(2 Marks)

(This exercise appeared in the first midterm exam in the Fall Term of 2012.)

Exercise 3.3 A Tricky Question using the Binomial Distribution

A mathematics textbook has 200 pages on which typographical errors in the equations could occur. Suppose there are in fact five errors randomly dispersed among these 200 pages.

- i) What is the probability that a random sample of 50 pages will contain at least one error?
(2 Marks)
- ii) How large must the random sample be to assure that at least three errors will be found with 90% probability? (You may use a normal approximation to the binomial distribution.)
(3 Marks)

(This exercise appeared in the first midterm exam in the Fall Term of 2012.)

Exercise 3.4

A system consists of two independent components connected in series. The life span (in hours) of the first component follows a Weibull distribution with $\alpha = 0.006$ and $\beta = 0.5$; the second has a lifespan in hours that follows the exponential distribution with $\beta = 25000$.

- i) Find the reliability of the system at 2500 hours.
(2 Marks)
- ii) Find the probability that the system will fail before 2000 hours.
(2 Marks)
- iii) If the two components are connected in parallel, what is the system reliability at 2500 hours?
(2 Marks)