

# Vv286 Honors Mathematics IV Ordinary Differential Equations

## Assignment 7

Date Due: 10:00 AM, Thursday, the 12<sup>th</sup> of November  
2015



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**Exercise 7.1.** Show that

$$\int_{-\infty}^{\infty} \frac{\cos x}{x^2 + a^2} dx = \pi \frac{e^{-a}}{a}, \quad \int_{-\infty}^{\infty} \frac{x \sin x}{x^2 + a^2} dx = \pi e^{-a}, \quad a > 0.$$

(3 + 3 Marks)

**Exercise 7.2.** Evaluate the integral

$$\int_{-\infty}^{\infty} \frac{dx}{1 + x^4}.$$

(3 Marks)

**Exercise 7.3.** Show that

$$\int_0^{\infty} \frac{x \sin x}{(x^2 + 4)^2} dx = \frac{\pi}{8e^2}.$$

(3 Marks)

**Exercise 7.4.** Show that

$$\int_{-\infty}^{\infty} \frac{dx}{(1 + x^2)^{n+1}} = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)} \pi, \quad n \in \mathbb{N}.$$

(3 Marks)

**Exercise 7.5.** Evaluate the following integrals using residue calculus:

$$\int_0^{\infty} \frac{\sqrt{x}}{x^2 + a^2} dx, \quad \int_0^{\infty} \frac{\ln x}{x^2 + a^2} dx, \quad a \in \mathbb{R}.$$

(2 + 3 Marks)

**Exercise 7.6.** If possible, use the Heaviside operator method to solve  $y'' + y = f(x)$  for

- i)  $f(x) = 3x + 5x^4,$
- ii)  $f(x) = e^{\mu x}, \mu \in \mathbb{R}.$

(3 + 3 Marks)

**Exercise 7.7.** Find the Laplace transform of the following functions:

- i)  $\sinh(bt), b \in \mathbb{R},$
- ii)  $\cos(bt), b \in \mathbb{R},$
- iii)  $t \sin(at), a \in \mathbb{R},$
- iv)  $t^2 \sinh(bt), b \in \mathbb{R},$
- v)  $\sqrt{t}$  (use the Euler gamma function).
- vi)  $1/\sqrt{t}$  (use the Euler gamma function).

(6 × 1 Mark)

**Exercise 7.8.** Use the Laplace transform to solve the following initial-value problems:

$$\begin{aligned}y''' - 6y'' + 11y' - 6y &= e^{4t}, & y(0) &= y'(0) = y''(0) = 0 \\y'' + y &= t \sin t, & y(0) &= 1, & y'(0) &= 2 \\y'' + y' + y &= 1 + e^{-t}, & y(0) &= 3, & y'(0) &= -5 \\y'' + y' + y &= H(t - \pi) - H(t - 2\pi), & y(0) &= 1, & y'(0) &= 0 \\y'' + y &= \begin{cases} \sin t & 0 \leq t < \pi \\ \cos t & \pi \leq t < \infty \end{cases}, & y(0) &= 1, & y'(0) &= 0 \\y'' + y &= \begin{cases} \cos t & 0 \leq t < \pi/2 \\ 0 & \pi/2 \leq t < \infty \end{cases}, & y(0) &= 3, & y'(0) &= -1\end{aligned}$$

You may use Laplace transform tables to look up the inverse transform.

(6 × 3 Marks)