International Journal of Control
Vol. 84, No. 4, April 2011, 758–765

Hard disk drive seeking profile optimisation via linear programming

Jun Zhang[ab]* and Xudong Wang[ab]

[a]*Department of Automation, Joint Institute of UM-SJTU, Shanghai Jiao Tong University, Shanghai, China;* [b]*Key Laboratory of System Control and Information Processing, Ministry of Education, Shanghai, China*

(*Received 10 August 2010; final version received 26 March 2011*)

In this article we develop an approach to design track-seeking controller for hard disk drive. We convert the current multi-sine seek approach to an equivalent linear programming problem that can be solved efficiently and accurately. We demonstrate the efficacy of this approach by numerical case studies that are otherwise infeasible in the current approach. We further derive the time optimal track seeking as a three-piece constant control. Our approach can also be extended to deal with other sophisticated requirements on smoothness, jerk and resonance.

Keywords: track seeking; multi-sine seek; linear programming; time optimal control

1. Introduction

With rapid technical advancements in materials, mechanics and electronics, the capacity of modern hard disk drive (HDD) has experienced an exponential growth in the past few decades (Daniel, Mee, and Clark 1998; Wang and Taratorin 1999). This renders a variety of challenging control problems, among which track following and track seeking of magnetic disk heads are the most crucial to the high performance of HDD. Track following aims at maintaining the disk head hovering above the centre of a track and thus attempts to achieve least variance control (Yi 2000; Chen, Lee, Peng, and Venkataramanan 2006; Mamun, Guo, and Bi 2007). On the other hand, track seeking strives to manoeuvre the head from one track to the other in the shortest time possible subject to all the physical constraints. Because of the distinct difference between these two controllers, they are usually separately designed and independently invoked. In this article, we will focus our attention on the servo controller design for track seeking.

Currently there are several different approaches to design track-seeking controllers. The dynamic model of magnetic disk head with voice coil motor (VCM) actuator can be described by a double integrator subject to bound constraints on motor voltage. In this case, it is well known that the time optimal control with bounded input is bang-bang control, i.e. the system can be steered from one state to the other by switching from one extremal value to the other once and only once (Bryson and Ho 1975). In engineering practices, bang-bang control is rarely used because it is sensitive to noises and model uncertainties and may cause significant resonance and extended settling time. An improvement is the proximate time optimal servomechanism (PTOS) proposed in Workman (1987) and Workman, Kosut, and Franklin (1987a, b), which provides a bumpless switching between one time optimal controller for large errors and one linear controller for small errors.

Multi-sine seek (MSS) is another widely used approach for track-seeking control design (Kang, Kim, and Chu 2005). It is a Fourier method, where the control input is written as the sum of a finite number of sinusoidal functions. Because sinusoidal functions form a basis for the Hilbert space of square integrable functions (Rudin 1986), they can be used to approximate any control input to an arbitrary accuracy if sufficiently many harmonics are engaged. Consequently, the track-seeking design problem on an infinitely dimensional function space has been converted to a constrained optimisation problem on a finite number of expansion coefficients. Fourier method is widely used in many scientific and engineering fields. For example, in the formal solution of partial differential equations, it is common to apply Fourier expansion and then turn the differential equation into algebraic equations on the expansion coefficients (Beerends, ter Morsche, van den Berg, and van de Vrie 2003).

The MSS design has the advantages of simple implementation and small online computational load for real-time embedded control. However, the resulting optimisation problem in the design stage has both

*Corresponding author. Email: jzhang@cal.berkeley.edu

ISSN 0020–7179 print/ISSN 1366–5820 online

DOI: 10.1080/00207179.2011.576432
http://www.informaworld.com

nonlinear cost function and highly complex nonlinear constrains. It is thus extraordinarily difficult to find numerical solutions for this problem.

In this article we will present an improvement of the current MSS design to overcome these drawbacks. We will show that the MSS design can be converted into an equivalent linear programming formulation and then be solved efficiently and accurately. We will further derive an analytic solution and show that the time optimal seeking is a three-piece constant control that switches between extremal values twice. The numerical solution with a large number of harmonics matches well with the theoretical solution. One additional boon of our approach is that it can be extended directly to deal with many other sophisticated performance requirements such as smoothness, jerk minimisation and resonance avoidance.

2. Current MSS formulation

To make this article self-contained, we first briefly review the current formulation of MSS controller design for HDD head track seeking (Kang et al. 2005; Dhanda, Xi, and Yu 2010).

The dynamic model of HDD head with VCM actuator under the back EMF effect can be described as

$$\ddot{x} = \dot{v} = K_a i, \tag{1}$$

$$u = L\dot{i} + Ri + K_e v, \tag{2}$$

where the variable x is the head position in the unit of track number, v the head velocity, i the VCM current, u the VCM voltage, the constant K_a is the acceleration constant, L the VCM coil inductance, R the VCM coil resistance and K_e the Back EMF constant. The control objective is to move the disk drive head from one track to the other in the shortest time, subject to the physical bounded constraints on motor voltage.

The MSS approach assumes that the seek profile of VCM current i is composed of a finite number of sinusoidal functions:

$$i^*(t) = \sum_{n=1}^{N} I_{c,n} \cos\frac{2n\pi}{T_{SK}}t + \sum_{n=1}^{N} I_{s,n} \sin\frac{2n\pi}{T_{SK}}t, \tag{3}$$

where T_{SK} is the total seeking time and $I_{c,n}$ and $I_{s,n}$ are the harmonic coefficients to be optimised. Integrating the VCM current i according to the model in Equation (1), we can get the head velocity and position as

$$v^*(t) = \sum_{n=1}^{N} \frac{K_a T_{SK}}{2n\pi} I_{s,n}\left(1 - \cos\frac{2n\pi}{T_{SK}}t\right) + \sum_{n=1}^{N} \frac{K_a T_{SK}}{2n\pi} I_{c,n} \sin\frac{2n\pi}{T_{SK}}t, \tag{4}$$

$$x^*(t) = \sum_{n=1}^{N} \frac{K_a T_{SK}}{2n\pi} I_{s,n} t - \sum_{n=1}^{N} \frac{K_a T_{SK}^2}{(2n\pi)^2} I_{s,n} \sin\frac{2n\pi}{T_{SK}}t + \sum_{n=1}^{N} \frac{K_a T_{SK}^2}{(2n\pi)^2} I_{c,n}\left(1 - \cos\frac{2n\pi}{T_{SK}}t\right). \tag{5}$$

Proceeding further, we can obtain the VCM voltage from Equation (2):

$$u(t) = \sum_{n=1}^{N} \frac{K_e K_a T_{SK}}{2n\pi} I_{s,n} + \sum_{n=1}^{N}\left(-\frac{L2n\pi}{T_{SK}} I_{c,n} + \frac{K_e K_a T_{SK}}{2n\pi} I_{c,n} + RI_{s,n}\right)\sin\frac{2n\pi}{T_{SK}}t + \sum_{n=1}^{N}\left(\frac{L2n\pi}{T_{SK}} I_{s,n} - \frac{K_e K_a T_{SK}}{2n\pi} I_{s,n} + RI_{c,n}\right)\cos\frac{2n\pi}{T_{SK}}t. \tag{6}$$

At the end of track seeking when $t = T_{SK}$, the following boundary condition has to be satisfied:

$$x^*(T_{SK}) = \sum_{n=1}^{N} \frac{K_a T_{SK}^2}{2n\pi} I_{s,n} = X_{SK}, \tag{7}$$

which implies that

$$T_{SK} = \sqrt{\frac{2\pi X_{SK}}{K_a \sum_{n=1}^{N} \frac{I_{s,n}}{n}}}. \tag{8}$$

Moreover, the VCM current i is desired to be zero at both initial and terminal time of seeking:

$$i^*(0) = i^*(T_{SK}) = \sum_{n=1}^{N} I_{c,n} = 0. \tag{9}$$

One physical constraint is that the VCM voltage u is restricted to the range of $[-V_s, V_s]$.

Now for a given seeking length X_{SK}, the current MSS design can be written as the following optimisation problem:

$$\min_{I_{s,n}, I_{c,n}} T_{SK} = \sqrt{\frac{2\pi X_{SK}}{K_a \sum_{n=1}^{N} \frac{I_{s,n}}{n}}}, \tag{10}$$

subject to

$$\sum_{n=1}^{N} I_{c,n} = 0, \tag{11}$$

$$\max_{0 \le t \le T_{SK}} |u(t)| \le V_s, \tag{12}$$

where $u(t)$ is given in Equation (6).

In this formulation of MSS design, control engineers are required to solve an optimisation problem with nonlinear cost function (10), equality constraint (11) and highly complex inequality constraint (12). Because of the nonlinear and non-convex nature of this optimisation problem, the current MSS design suffers several serious drawbacks. First, the resulting optimisation problem is so complicated that it consumes a significant amount of computational time in the design stage. Even worse, the numerical procedure can be easily trapped into local minima and fails to find a satisfactory solution. Second, it also restricts the harmonics that can be engaged in the MSS design to a small number such as 4 or 5, and thus prohibits it from achieving a higher accuracy, as a large number of harmonics will cost an unbearable computer time. Third, it is virtually infeasible to incorporate any other performance requirements, as the resulting optimisation problem becomes even more complicated and cannot be solved numerically.

3. MSS design via linear programming

To remedy these difficulties, we present an improvement of the current MSS design by converting it into an equivalent linear programming problem, which can then be solved efficiently and accurately. We consider an inverse problem, that is, finding the maximum seeking length X_{SK} for a given seeking time T_{SK}. In other words, we want to find out how far the drive head can seek within a certain time duration. Notice that when T_{SK} is a monotonic function of X_{SK}, which is indeed the case here, this is equivalent to the original problem.

Given a seeking time T_{SK}, we can rewrite the seek length X_{SK} in Equation (7) as

$$X_{SK} = f^T x, \tag{13}$$

where

$$f = \frac{K_a T_{SK}^2}{2\pi}\left[1, \frac{1}{2}, \ldots, \frac{1}{N}, 0, \ldots, 0\right]^T \in \mathbb{R}^{2N}, \tag{14}$$

and $x = [I_{s,1}, \ldots, I_{s,N},\ I_{c,1}, \ldots, I_{c,N}] \in \mathbb{R}^{2N}$ is a vector composed of the Fourier expansion coefficients to be optimised.

Divide the total seeking time interval $[0, T_{SK}]$ into M small intervals $\{[t_k, t_{k+1}]\}_{k=0}^{M-1}$, where $t_0 = 0$ and $t_M = T_{SK}$. When M is large enough, we can use piecewise constant functions to approximate the VCM voltage $u(t)$. In this case, the constraint in Equation (12) can be written as

$$-V_s \le u(t_k) \le V_s, \quad k = 0, \ldots, M. \tag{15}$$

From Equation (6), we obtain the VCM voltage $u(t_k)$ as

$$\begin{aligned} u(t_k) = {} & \sum_{n=1}^{N}\left(R\sin\frac{2n\pi}{T_{SK}}t_k + \frac{K_eK_aT_{SK}}{2n\pi} + \frac{L2n\pi}{T_{SK}}\cos\frac{2n\pi}{T_{SK}}t_k \right. \\ & \left. - \frac{K_eK_aT_{SK}}{2n\pi}\cos\frac{2n\pi}{T_{SK}}t_k\right)I_{s,n} \\ & + \sum_{n=1}^{N}\left(R\cos\frac{2n\pi}{T_{SK}}t_k - \frac{L2n\pi}{T_{SK}}\sin\frac{2n\pi}{T_{SK}}t_k \right. \\ & \left. + \frac{K_eK_aT_{SK}}{2n\pi}\sin\frac{2n\pi}{T_{SK}}t_k\right)I_{c,n}. \end{aligned} \tag{16}$$

Letting $H_k = [H_k^{11}, \ldots, H_k^{1N}, H_k^{21}, \ldots, H_k^{2N}]^T$, where

$$\begin{aligned} H_k^{1n} = {} & R\sin\frac{2n\pi}{T_{SK}}t_k + \frac{K_eK_aT_{SK}}{2n\pi} + \frac{L2n\pi}{T_{SK}}\cos\frac{2n\pi}{T_{SK}}t_k \\ & - \frac{K_eK_aT_{SK}}{2n\pi}\cos\frac{2n\pi}{T_{SK}}t_k, \end{aligned} \tag{17}$$

$$\begin{aligned} H_k^{2n} = {} & R\cos\frac{2n\pi}{T_{SK}}t_k - \frac{L2n\pi}{T_{SK}}\sin\frac{2n\pi}{T_{SK}}t_k \\ & + \frac{K_eK_aT_{SK}}{2n\pi}\sin\frac{2n\pi}{T_{SK}}t_k, \end{aligned} \tag{18}$$

we can rewrite $u(t_k)$ in a more compact form:

$$u(t_k) = H_k^T x. \tag{19}$$

Consequently, the constraint equation (15) can be written in the form of linear matrix inequalities on the optimisation variable x:

$$-V_s \le H_k^T x \le V_s, \quad k = 0, \ldots, M. \tag{20}$$

Now it is straightforward that the MSS profile design can be casted into the following linear programming problem:

$$\max_x X_{SK} = f^T x, \tag{21}$$

subject to

$$c_1^T x = 0, \tag{22}$$

$$-V_s \le H_k^T x \le V_s, \quad k = 0, \ldots, M, \tag{23}$$

where $c_1 = [0, \ldots, 0, 1, \ldots, 1]^T$. Note that the constraint (22) is obtained directly from Equation (11).

We have now converted the original MSS profile design into an equivalent linear programming problem (Equations (21)–(23)). It is well-known that the numerical procedures for solving linear programming are mature and easily accessible (Polak 1997). We can thus solve the MSS design for track seeking with high efficiency and accuracy.

4. Numerical studies

In this section, we will investigate several different seeking scenarios and also show that our improved design approach is efficient in solving optimal profile design even with several hundreds harmonics.

We adopt model parameters from a prototype HDD product (Yi 2000). According to the seeking length, the MSS profile design can be differentiated into three cases: short, medium and long seeking. For each case, we start from the old MSS profile design method with $N=4$, i.e. four harmonic terms. We then apply our improved design approach to solve for the cases when $N=4$, 10, 20, 40 and 100. Note that when N is bigger than 4, the old design method is not able to find a satisfactory solution. The saving in computational time is considerable: even when there are 100 harmonics, the total computational time is several minutes on a mundane personal desktop computer. These numerical results are shown in Figure 1, where we plot the given seeking time versus the obtained maximum seeking length for these three cases.

Taking a closer look at these numerical results, we can draw the following conclusions on seeking performance:

(1) The result of $N=10$ has a clear improvement over $N=4$;
(2) When $N>20$, the performance enhancement is marginal, especially for the short seek;
(3) The difference between $N=40$ and $N=100$ is negligible, we thus contemplate that $N=100$ is close to the theoretical limit.

In Figure 2, we plot the voltage, current and velocity profiles for different choices of N for the case when the seeking time $T_{SK}=128$ samples in long seeking. Notice that the voltage begins with a value close to its maximum, then smoothly switches to the minimum and finally goes back to its maximum. This is clearly an approximation to a Bang-Bang type of control.

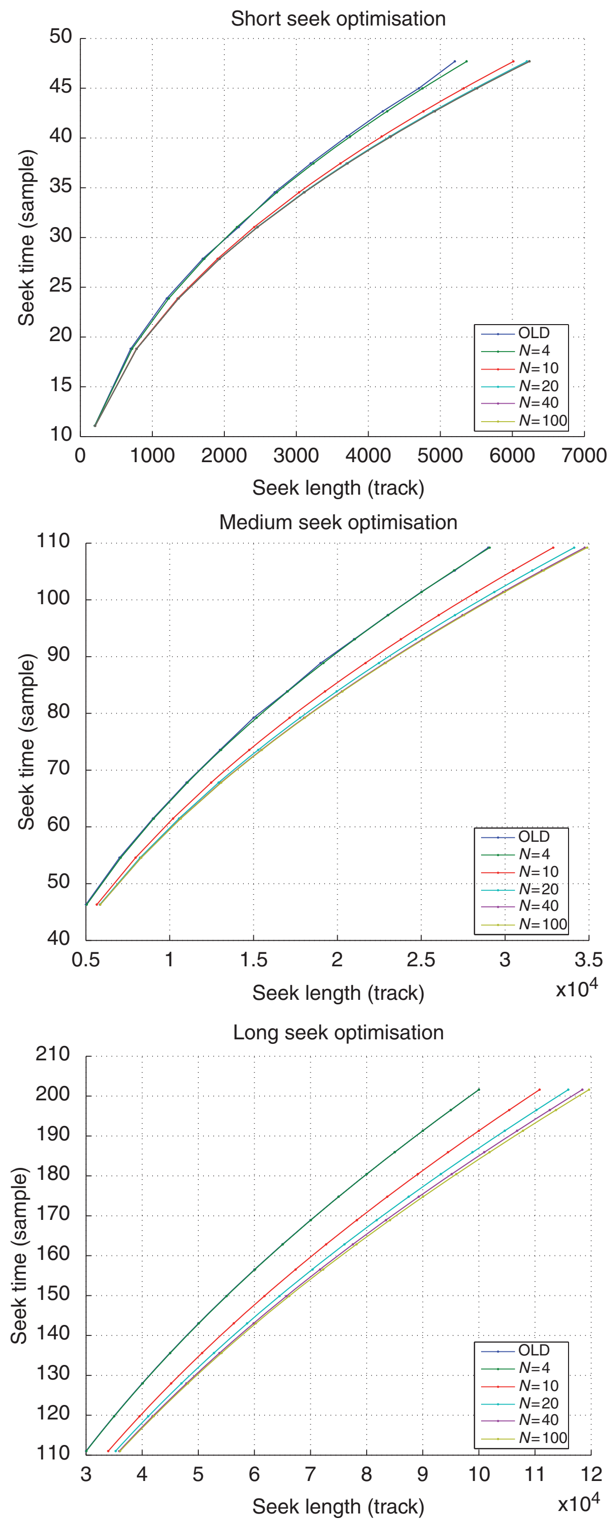

Figure 1. Short, medium and long seeking optimisation results: x-axis is the obtained maximal seeking length and y-axis the seeking time. Available in colour online.

5. Theoretical analysis

We have formulated the MSS profile design into a linear programming problem and then demonstrated its efficacy by numerical studies. We observed that when a large number of harmonics are employed, the resulting optimal VCM voltage tends to approach a Bang-Bang type of control. Inspired by this observation, we will derive the theoretical solution of time optimal control in this section.

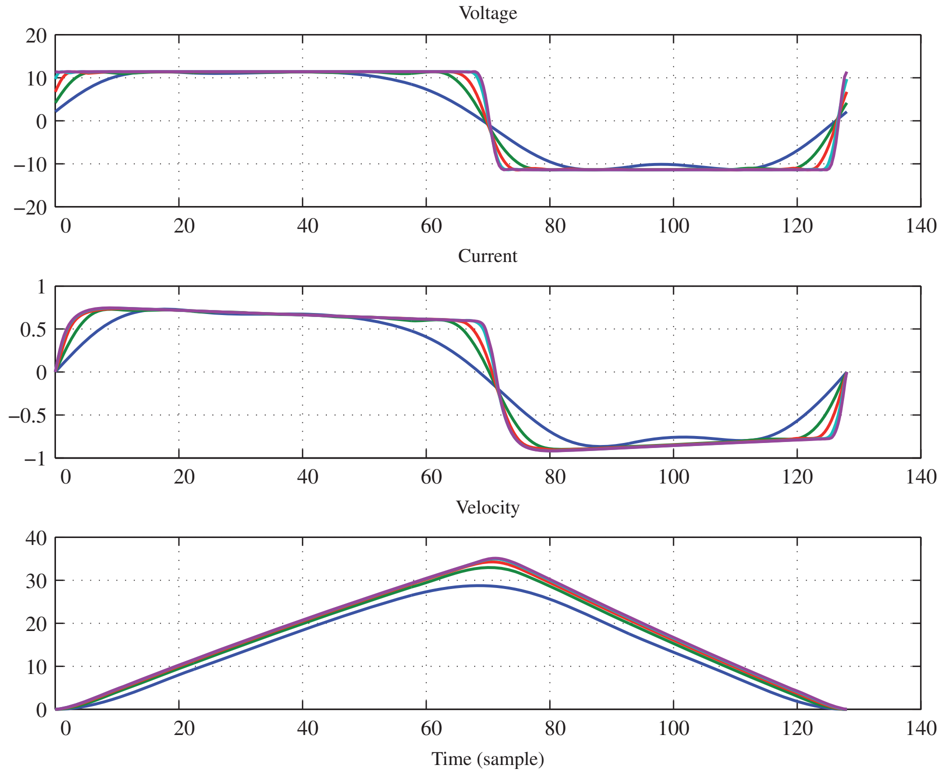

Figure 2. Optimal voltage, current and velocity profiles for a given seeking time $T_{SK} = 128$ samples.

For the ease of analysis, we rewrite the dynamic model in Equation (1) in a state space form:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} x \\ v \\ i \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & K_a \\ 0 & -\dfrac{K_e}{L} & -\dfrac{R}{L} \end{pmatrix}\begin{pmatrix} x \\ v \\ i \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dfrac{1}{L} \end{pmatrix} u. \quad (24)$$

The control objective is to steer the head position x from one track to the other in the shortest time. The cost index can then be written as

$$\min \int_0^t 1 \, \mathrm{d}\tau, \quad (25)$$

and the control input of VCM voltage $u(t)$ is constrained in $[-V_s, V_s]$. Construct the Hamiltonian as

$$H = 1 + l_1 v + l_2 K_a i + l_3 \left(-\frac{K_e}{L} v - \frac{R}{L} i \right) + \frac{l_3}{L} u, \quad (26)$$

where l_1, l_2 and l_3 are costate variables. Pontryagin's maximal principle asserts that the optimal control $u(t)$ is given in the form of bang-bang control (Bryson and Ho 1975):

$$u(t) = \begin{cases} V_s, & \text{if } l_3 > 0; \\ -V_s, & \text{if } l_3 < 0. \end{cases} \quad (27)$$

Notice that in the model (24) the position variable x only influences the dynamics of the velocity v. We can then study a reduced system

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} v \\ i \end{pmatrix} = \begin{pmatrix} 0 & K_a \\ -\dfrac{K_e}{L} & -\dfrac{R}{L} \end{pmatrix}\begin{pmatrix} v \\ i \end{pmatrix} + \begin{pmatrix} 0 \\ \dfrac{1}{L} \end{pmatrix} u. \quad (28)$$

For the parameters K_a, K_e, R and L from our prototype model, the system matrix

$$\begin{pmatrix} 0 & K_a \\ -\dfrac{K_e}{L} & -\dfrac{R}{L} \end{pmatrix} \quad (29)$$

has two real eigenvalues λ_1 and λ_2:

$$\lambda_1 = \frac{-R + \sqrt{R^2 - 4LK_aK_e}}{2L}, \quad (30)$$

$$\lambda_2 = \frac{-R - \sqrt{R^2 - 4LK_aK_e}}{2L}, \quad (31)$$

and the corresponding eigenvectors are

$$\begin{pmatrix} K_a \\ \lambda_1 \end{pmatrix} \text{ and } \begin{pmatrix} K_a \\ \lambda_2 \end{pmatrix}. \quad (32)$$

From the control law (27), the VCM voltage $u(t)$ switches between two boundary values based on the sign of l_3. To find out the switching strategy, we need to solve for the integral curve of Equation (28). Apply a coordinate transformation on Equation (28) by letting $z = T^{-1}[v, i]^T$, where the transformation matrix T is defined as

$$T = \begin{pmatrix} K_a & K_a \\ \lambda_1 & \lambda_2 \end{pmatrix}. \quad (33)$$

We then obtain a new set of differential equations:

$$\dot{z}_1 = \lambda_1 z_1 - \frac{1}{(\lambda_2 - \lambda_1)L} u, \quad (34)$$

$$\dot{z}_2 = \lambda_1 z_2 + \frac{1}{(\lambda_2 - \lambda_1)L} u. \quad (35)$$

Proceeding further, we get

$$\frac{\mathrm{d}z_1}{\mathrm{d}z_2} = \frac{\lambda_1 z_1 - \frac{1}{(\lambda_2 - \lambda_1)L} u}{\lambda_2 z_2 + \frac{1}{(\lambda_2 - \lambda_1)L} u}. \quad (36)$$

After some algebraic derivations, we can derive that the integral curve of Equations (34) and (35) for constant VCM voltage u is given by

$$e^{z_1(0)} \left(\lambda_1 z_1 - \frac{1}{(\lambda_2 - \lambda_1)L} u \right)^{\frac{1}{\lambda_1}} = e^{z_2(0)} \left(\lambda_2 z_2 + \frac{1}{(\lambda_2 - \lambda_1)L} u \right)^{\frac{1}{\lambda_2}}. \quad (37)$$

Transforming back into the original (v, i) coordinates, we have the integral curve for Equation (28) as

$$\left(\lambda_1 \frac{\lambda_2 v - K_a i}{(\lambda_2 - \lambda_1)K_a} - \frac{1}{(\lambda_2 - \lambda_1)L} u \right)^{\frac{1}{\lambda_1}} = C \left(\lambda_2 \frac{-\lambda_1 v + K_a i}{(\lambda_2 - \lambda_1)K_a} + \frac{1}{(\lambda_2 - \lambda_1)L} u \right)^{\frac{1}{\lambda_2}}, \quad (38)$$

where C is some constant.

From Equation (38), we can now plot integral curves of the dynamic model (28) in the phase plane, as shown in Figure 3. The blue solid line corresponds to the case when the maximal voltage is applied, and the red dashed line the minimal voltage. For track seeking, it is required that the velocity v and the current i are both zero at the initial and terminal time instants. In the phase plane, this implied that the optimal trajectory starts from the origin and goes back to the origin at the end. From phase trajectories, we can conclude that for time optimal seeking, the voltage u has to switch twice and thus is a three-piece constant function:

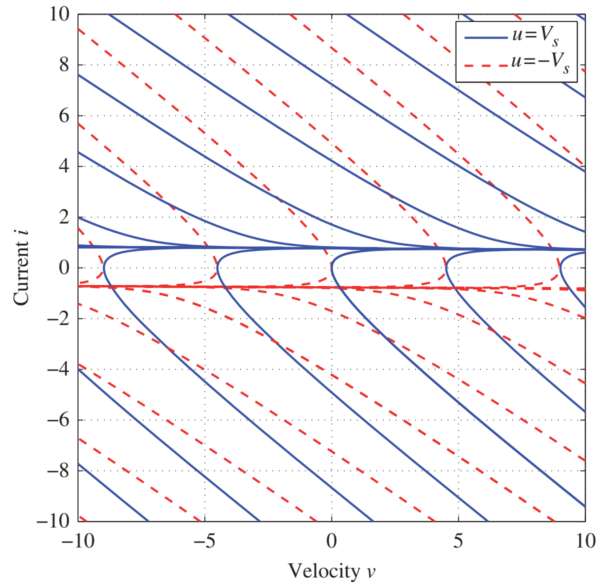

Figure 3. Planar integral curves for the head velocity v and VCM current i. Blue solid: $u(t) = V_s$; Red dashed: $u(t) = -V_s$. Available in colour online.

(1) The voltage u starts at the maximal value V_s and the phase trajectory leaves the origin by following a blue solid integral curve;
(2) It switches to the minimal value $-V_s$ at $t = t_1$ and follows a red dashed curve;
(3) It switches back to the maximal value V_s at $t = t_2$ and goes back to the origin along another blue solid curve.

For better illustration, we also plot a 3D integral curve in the phase space for the position x, the head velocity v and the VCM current i in Figure 4. Note that here the switching time instants t_1 and t_2 are determined solely by the seeking length X_{SK}, and they can be solved by numerical procedures efficiently.

From the theoretical analysis, it is clear that the MSS seeking profile tends to time optimal control when sufficiently many harmonic terms are employed. This result is made possible only by our improved design approach, as it can easily deal with a large number of harmonics.

6. Conclusions

In this article, we developed a new approach to design track-seeking controller for the HDD. We showed that the current MSS profile optimisation can be converted into a linear programming problem and thus be solved

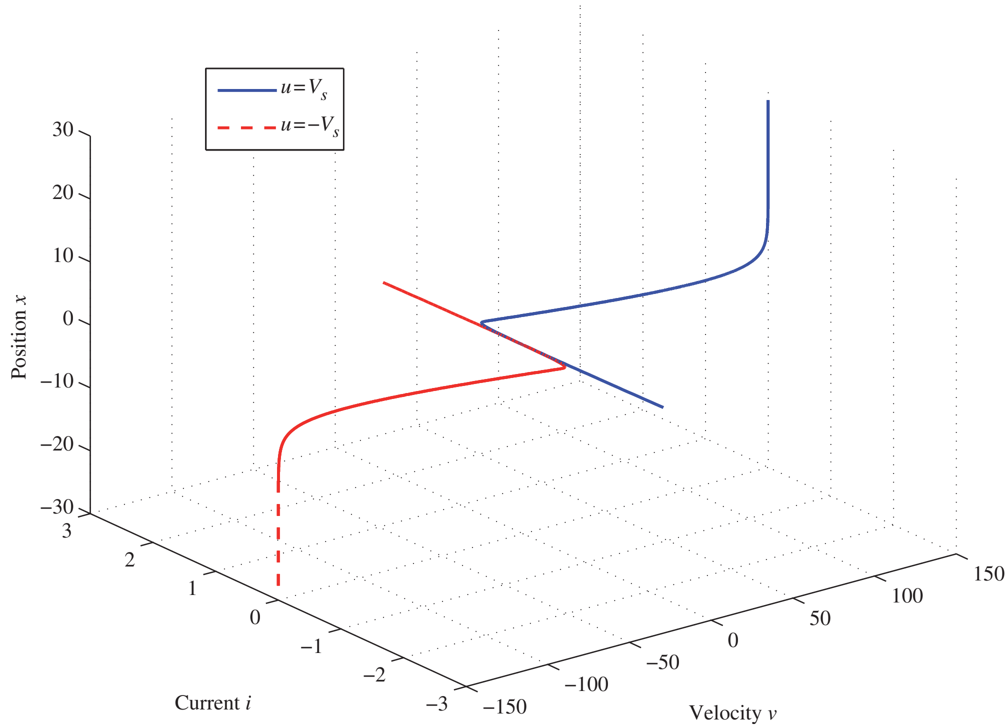

Figure 4. A 3D integral curve for the position x, head velocity v, and VCM current i. Blue solid: $u(t) = V_s$; Red dashed: $u(t) = -V_s$. Available in colour online.

with high efficiency and accuracy. This new approach overcomes the numerical difficulties in the current method and provides satisfactory control design. We also derived the time optimal seeking profile as a three-part piecewise constant control, which turns out to be the performance limit when the number of harmonic terms goes to infinity.

We want to point out that in real applications there are some additional engineering concerns that have to be taken into account. First, we often want to avoid certain resonant modes due to the HDD mechanical design. Second, it is usually desired to reduce acoustic noises for home users, which is then translated into the requirements on smoothness and jerk minimisation. In these cases, the time optimal seeking cannot be applied directly, because this will bring in rich harmonics that could cause significant resonances. Moreover, it may also consume more energies and cause higher acoustic noises.

Our new approach can be extended to deal with these engineering complicacies by directly encoding additional cost function terms and linear matrix inequality constraints. For example, to avoid certain mechanical resonant mode, we can remove the harmonic terms close to the resonant frequencies in the optimisation formulation. The resulting optimisation problem can still be solved with high efficiency. All these illustrate that our approach is effective and flexible for various design purposes in HDD track-seeking control design.

Acknowledgements

Jun Zhang thanks the Innovation Program of Shanghai Municipal Education Commission for financial support under Grant No. 11ZZ20.

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