Study on the closed-loop properties of GPC*

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Abstract Based on the coefficient mapping of eigenpolynomial from plant to GPC closed-loop system in the IMC structure, the solvability of the optimal control law of GPC and some properties of GPC in relationship with tuning parameters are discussed. Theorems on GPC resulting in deadbeat control are given by properly choosing the design parameters. Furthermore, new result about reducing GPC closed-loop system is derived, which extends the former conclusions. It also provides a new way to study the properties of GPC system based on coefficient mapping.

Keywords: **generalized predictive control, internal model control, solvability, deadbeat control, reducedorder system.**

In recent years, model predictive control (MPC) seems to be one of the most attractive advanced process control algorithms. The combination of new control design concepts in MPC, such as model prediction, receding horizon optimization and real-time correction, makes it possible to yield high performance for control systems. Among various MPC algorithms, generalized predictive control (GPC)[1] has received particular attention. However, in contrast to the rapid development of MPC in application areas, the theoretical study of MPC properties seems still scarce. Only a small number of studies have been focused on the closed-loop properties of GPC and other MPC algorithms in relationship with the tuning parameters[2—5]. Among these, excellent results have been achieved by Clarke and Mohtadi[5]. In the form of LQ problem, some new results on the GPC properties such as deadbeat control and stability were presented.

Eigenpolynomial always plays a key role in studying the dynamic characteristics of the system. Some researchers have also developed the closed-loop transfer function for MPC to analyze its closed-loop properties[3,6]. However, the coefficients of the eigenpolynomials are sometimes implicit and considered as the results of some recursive equations so that they cannot be easily used for system analysis. In this paper, based on the coefficient mapping of eigenpolynomial from plant to GPC closed-loop system in the IMC structure[7], the solvability of the optimal control law of GPC and some properties of GPC in relationship with tuning parameters are discussed. Theorems on GPC resulting in deadbeat control are given by properly choosing the design parameters. Furthermore, new result about reducing GPC closed-loop is derived, which extends the former conclusions. It provides a new way to study the properties of GPC system based on coefficient mapping.

1 GPC closed-loop description in IMC structure

Firstly, GPC closed-loop description in IMC structure is presented concisely. GPC is based

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on the following CARIMA model:

$$A(z^{-1})y(t) = B(z^{-1})u(t-1) + \xi(t)/\Delta, \ \Delta = 1 - z^{-1}, \tag{1}$$

where $u(t)$ is the control input, $y(t)$ the output, and $\xi(t)$ an uncorrelated random sequence; A and B are the polynomials in the backward shift operator z^{-1}:

$$B(z^{-1}) = m_1 + m_2 z^{-1} + \cdots + m_n z^{-n+1}, \tag{2}$$

$$A(z^{-1}) = 1 + p_1 z^{-1} + \cdots + p_n z^{-n}. \tag{3}$$

It is generally assumed that A and B are irreducible.

Consider the cost function of the form

$$J(t) = \mathrm{E}\left\{ \sum_{j=N_1}^{N_2} [y(t+j) - \omega(t+j)]^2 + \sum_{j=1}^{N_u} \lambda [\Delta u(t+j-1)]^2 \right\}. \tag{4}$$

GPC optimal control law can be given by[1]

$$\Delta u(t) = \boldsymbol{d}^{\mathrm{T}}(\boldsymbol{w} - \boldsymbol{Y}_{\mathrm{p}}), \tag{5}$$

where

$$\boldsymbol{d}^{\mathrm{T}} = (1\ 0\ \cdots\ 0)(G^{\mathrm{T}}G + \lambda I)^{-1}G^{\mathrm{T}} \triangleq (d_1 \cdots d_{N_2-N_1+1}), \tag{6}$$

$\boldsymbol{Y}_{\mathrm{p}}$ denotes the free responses of the process and G the matrix which consists of step responses $\{a_i\}$ of the plant:

$$G = \begin{bmatrix} a_{N_1} & \cdots & 0 & \\ & & \ddots & \\ \vdots & & & 0 \\ & & & \vdots \\ a_{N_2} & & \cdots & a_{N_2-N_u+1} \end{bmatrix}, \ \boldsymbol{w} = \begin{bmatrix} \omega(t+N_1) \\ \vdots \\ \omega(t+N_2) \end{bmatrix}.$$

In order to analyze the properties of the GPC system, no model mismatch and disturbance are assumed as in ref. [5]. According to the IMC description of GPC in ref. [8], the closed-loop transfer function of the GPC system can be given by

$$G(z^{-1}) = \frac{d_s z^{-1} B(z^{-1})}{A_c(z^{-1})}, \tag{7}$$

where

$$d_s = \sum_{i=1}^{N_2-N_1+1} d_i, \ A_c(z^{-1}) = 1 + p_1^* z^{-1} + \cdots + p_{n+1}^* z^{-(n+1)}$$

The coefficients of $A_c(z^{-1})$ are determined by

$$\begin{bmatrix} 1 \\ p_1^* \\ \vdots \\ p_{n+1}^* \end{bmatrix} = \begin{bmatrix} 1 & & & 0 \\ b_{N_1+1} - 1 & 1 & & \\ \vdots & \ddots & \ddots & \\ \vdots & & \ddots & 1 \\ b_{N_1+n+1} - b_{N_1+n} & \cdots\cdots & & b_{N_1+1} - 1 \end{bmatrix} \begin{bmatrix} 1 \\ p_1 \\ \vdots \\ p_n \end{bmatrix}, \tag{8}$$

and

$$b_i = \sum_{j=1}^{N_2-N_1+1} d_j a_{i+j-1}.$$

Equation (8) shows the coefficient mapping from the eigenpolynomial of the plant to GPC closed-loop system. It provides the foundation for analyzing the closed-loop characteristics in this paper. For the need of further study, several lemmas describing the basic properties of eigenpolynomial coefficients mapping of eq. (8) are given first.

Lemma 1.1. *Let the plant be described by*

$$G_p(z^{-1}) = \frac{z^{-1}B(z^{-1})}{A(z^{-1})}, \tag{9}$$

where A, B are given in eqs. (2), (3). Then the step responses $\{a_i\}$ and the coefficients of A, B satisfy

$$\begin{cases} a_1 = m_1, \\ (a_2 - a_1) + a_1 p_1 = m_2, \\ \quad\vdots \\ (a_n - a_{n-1}) + (a_{n-1} - a_{n-2}) p_1 + \cdots + a_1 p_{n-1} = m_n, \\ (a_{n+1} - a_n) + (a_n - a_{n-1}) p_1 + \cdots + a_1 p_n = 0, \\ (a_{i+2} - a_{i+1}) + (a_{i+1} - a_i) p_1 + \cdots + (a_{i-n+2} - a_{i-n+1}) p_n = 0, \ i \geqslant n. \end{cases} \tag{10}$$

Lemma 1.2. *If $\lambda = 0$ and G is full rank in column, then we have*

$$\begin{cases} b_{N_1} = 1, \\ b_{N_1-1} = \cdots = b_{N_1-N_u+1} = 0. \end{cases} \tag{11}$$

Proof. Set $\lambda = 0$ in eq. (6) and right-multiply G on both sides of eq. (6), we have

$$\begin{cases} d_1 a_{N_1} + \cdots + d_{N_2-N_1+1} a_{N_2} = 1, \\ d_1 a_{N_1-1} + \cdots + d_{N_2-N_1+1} a_{N_2-1} = 0, \\ \quad\vdots \\ d_1 a_{N_1-N_u+1} + \cdots + d_{N_2-N_1+1} a_{N_2-N_u+1} = 0, \end{cases}$$

where $a_j = 0$, $\forall j \leqslant 0$. Then the lemma follows.

$$\begin{bmatrix} m_1 \\ \vdots \\ m_n \\ 0 \\ 0 \\ \vdots \end{bmatrix} = \begin{bmatrix} a_1 & 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & & \vdots \\ a_n & \cdots & a_1 & 0 & 0 \\ a_{n+1} & \cdots & \cdots & a_1 & 0 \\ a_{n+2} & \cdots & \cdots & \cdots & a_1 \\ \vdots & & & & \vdots \end{bmatrix} \begin{bmatrix} 1 \\ p_1 - 1 \\ \vdots \\ p_n - p_{n-1} \\ -p_n \end{bmatrix}. \tag{12}$$

For $N_u \geqslant n+2$ and $N_1 \geqslant n+1$, we have

$$\begin{bmatrix} 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \end{bmatrix} = \underbrace{\begin{bmatrix} a_{n+1} & \cdots a_1 & & 0 \\ \vdots & \vdots & \ddots & \\ a_{N_u} & \cdots & a_{N_u - n} \cdots & a_1 \\ \vdots & & \vdots & \vdots \\ \underbrace{a_{N_2} \quad \cdots \quad a_{N_2 - n}}_{n+1} & & \underbrace{\cdots \quad a_{N_2 - N_u + 1}}_{N_u - (n+1)} \end{bmatrix}}_{G^*} \begin{bmatrix} 1 \\ p_1 - 1 \\ \vdots \\ p_n - p_{n-1} \\ -p_n \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

Obviously, G is a part of G^* (the last $N_2 - N_1 + 1$ rows of G^*). Therefore the column vectors of G are correlated. Then there exists no unique optimal control law of GPC.

Theorem 2.4. *To the nth order plant, if* $N_u \geqslant n+2$, $N_1 \leqslant n$, $N_2 \geqslant N_u + n - 1$ and $m_n \neq 0$, *there must exist a unique optimal control law of GPC.*

Proof. When $N_u \geqslant n+2, N_1 \leqslant n, N_2 \geqslant N_u + N_1 - 1$,

$$G = \begin{bmatrix} a_{N_1} & \cdots a_1 & 0 & \cdots\cdots & 0 \\ \vdots & & \ddots & \ddots & \vdots \\ a_n & \cdots\cdots\cdots & a_1 & \ddots & \vdots \\ \vdots & & & \ddots & 0 \\ a_{N_u} & \cdots\cdots\cdots\cdots\cdots & & & a_1 \\ \vdots & & & & \vdots \\ a_{N_2} & \cdots\cdots\cdots\cdots\cdots & & & a_{N_2 - N_u + 1} \end{bmatrix}.$$

Using relationship (12), we can make column transformation as follows:

Lemma 1.3. *In GPC, we always have* $1+p_1^*+\cdots+p_{n+1}^*=d_sB(1)$.

Proof. Multiplying both sides of eq. (8) by $(1\cdots 1)$, we get

$$1+p_1^*+\cdots+p_{n+1}^*=(b_{N_1+n+1}\ b_{N_1+n}\cdots b_{N_1+1})\boldsymbol{p}$$

$$=(d_1\cdots d_{N_2-N_1+1})\begin{pmatrix} a_{N_1+n+1} & a_{N_1+n} & \cdots & a_{N_1+1} \\ \vdots & \vdots & & \vdots \\ a_{N_2+n+1} & a_{N_2+n} & \cdots & a_{N_2+1} \end{pmatrix}\begin{pmatrix} 1 \\ p_1 \\ \vdots \\ p_n \end{pmatrix},$$

and summing up the first $n+i-1$ equations of (10), we have

$$a_{i+n-1}+a_{i+n-2}p_1+\cdots+a_{i-1}p_n=m_1+\cdots+m_n=B(1),\ i\geqslant 1.$$

Then it follows that $1+p_1^*+\cdots+p_{n+1}^*=d_sB(1)$.

2 Solvability of optimal control law

In this paper, solvability means that there exists a unique solution to cost function (4). The optimal control law given in eq. (5) assumes its existence, but it must be checked if a unique solution can really be found. It is meaningful to discuss the solvability of the optimization problem. From eq. (6), it is clear that the existence and uniqueness of the control law are guaranteed by the non-singularity of the matrix $(G^TG+\lambda I)$. So only the rank of the matrix $(G^TG+\lambda I)$ needs to be discussed. First consider the case of $\lambda>0$.

Theorem 2.1. *If* $\lambda>0$, *there must exist a unique optimal control law of GPC.*

This result can be easily obtained, so non-zero control weighting can always guarantee a unique optimal control law. However, optimal solution may not exist when control increments are not penalized in the cost function. So we will focus on the case $\lambda=0$. Considering that the matrix (G^TG) is non-singular if and only if G has full rank in column, we should only study the column rank of G.

Theorem 2.2. *To the nth order plant, if* $N_u\leqslant n+1$, $N_2\geqslant N_1+N_u-1$, *there exists a unique optimal control law of GPC.*

It can be derived by the property of Markov parameters directly.

Theorem 2.3. *To the nth order plant, if* $N_u\geqslant n+2$, $N_1\geqslant n+1$, *there exists no unique optimal control law of GPC.*

Proof. Eq. (10) can be rewritten as

$$G \xrightarrow[\cdots + (n+2)*(-p_n)]{(1) \leftarrow (2)*(p_1 - 1) +} \begin{bmatrix} m_{N_1} & a_{N_1-1} & \cdots & a_1 & 0 & \cdots\cdots & 0 \\ \vdots & \vdots & & \ddots & \ddots & & \vdots \\ m_n & a_{n-1} & \cdots\cdots & & a_1 & 0 & \vdots \\ 0 & a_n & \cdots\cdots & & & a_1 \ddots & \vdots \\ \vdots & \vdots & & & & \ddots & 0 \\ 0 & a_{N_u-1} & \cdots\cdots & & & & a_1 \\ \vdots & \vdots & & & & & \vdots \\ 0 & a_{N_2-1} & \cdots\cdots & & & & a_{N_2-N_u+1} \end{bmatrix} \rightarrow$$

$$\cdots \rightarrow \begin{bmatrix} \overbrace{* \ \cdots \ *}^{N_u - n} & & & \\ \vdots \quad\ \ \vdots & & & \\ * \quad\ \ \vdots & & * & \\ m_n \ \ddots \ \vdots & & & \\ \ddots \ * & & & \\ 0 \quad m_n & & & \\ & a_n & \cdots & a_1 \\ 0 & \vdots & & \vdots \\ & a_{N_2-N_u+n} & \cdots & a_{N_2-N_u+1} \end{bmatrix}.$$

Since $\begin{bmatrix} a_n & \cdots & a_1 \\ \vdots & & \vdots \\ a_{N_2-N_u+n} & \cdots & a_{N_2-N_u+1} \end{bmatrix}$ is full rank in column and $m_n \neq 0$, the column rank of G is N_u, and there must exist a unique optimal control law of GPC.

The above theorems discuss the solvability issue in all the cases of λ, N_1 and N_u completely. It should be noted that N_2 can be selected as $N_2 \geqslant N_1 + N_u - 1$ just to satisfy the necessary condition of G full rank in column. All these discussions could help to choose the proper tuning parameters and guarantee the further discussion meaningful.

3 Deadbeat control property in GPC

Deadbeat control always has the transfer function of the form

$$G(z^{-1}) = l_1 z^{-1} + \cdots + l_n z^{-n}. \tag{13}$$

It is clear that the impulse response of the system will become zero after n sampling periods. From eqs. (7) and (13) it is clear that, if all the closed-loop poles are placed at the origin, the GPC system becomes a dead-beat one. Therefore, a necessary and sufficient condition for GPC as deadbeat control is $A_c(z^{-1}) = 1$, which is equivalent to $p_1^* = \cdots = p_{n+1}^* = 0$.

The following corollary can be easily derived.

Corollary 3.1. *To the nth order plant, if $\lambda = 0$ and G is full rank in column, then* $p_{n+1}^* = 0$.

Proof. According to Lemma 1.2,

$$b_{N_1} = d_1 a_{N_1} + \cdots d_{N_2-N_1+1} a_{N_2} = 1,$$

$$\therefore p_{n+1}^* = [b_{N_1+n+1} - b_{N_1+n} \cdots b_{N_1+1} - 1] \boldsymbol{p}$$

$$= [d_1 \cdots d_{N_2-N_1+1}] \begin{bmatrix} a_{N_1+n+1} - a_{N_1+n} \cdots a_{N_1+1} - a_{N_1} \\ \vdots \qquad\qquad \vdots \\ a_{N_2+n+1} - a_{N_2+n} \cdots a_{N_2+1} - a_{N_2} \end{bmatrix} \begin{bmatrix} 1 \\ p_1 \\ \vdots \\ p_n \end{bmatrix}.$$

Using Lemma 1.1, $p_{n+1}^* = 0$, and then the closed-loop system will be of the nth order.

The above corollary gives the condition for p_{n+1}^* equaling zero. Furthermore, the condition for any p_i^* equaling zero will be given.

Theorem 3.1. *To the nth order plant, if $\lambda = 0$ and G is full rank in column, and* $N_1 \geqslant n + 1 - i$, $N_u \geqslant n + 2 - i$, *then* $p_i^* = 0$, $1 \leqslant i \leqslant n$.

Proof. Under the above condition, Lemma 1.2 holds.

$$p_i^* = [b_{N_1+i} - b_{N_1+i-1} \cdots 1 \overbrace{0 \cdots 0}^{n-i}] \boldsymbol{p}.$$

$$\because N_u \geqslant n + 2 - i, \therefore N_1 + i - n - 1 \geqslant N_1 - N_u + 1,$$

then $b_{N_1} = 1$ and $b_{N_1-1} = \cdots = b_{N_1+i-n-1} = 0$, so

$$p_1^* = (b_{N_1+i} - b_{N_1+i-1} \cdots b_{N_1} - b_{N_1-1} \cdots b_{N_1+i-n} - b_{N_1+i-n-1}) \boldsymbol{p}$$

$$= (d_1 \cdots d_{N_2-N_1+1}) \begin{pmatrix} a_{N_1+i} - a_{N_1+i-1} \cdots a_{N_1} - a_{N_1-1} \cdots a_{N_1+i-n} - a_{N_1+i-n-1} \\ \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ a_{N_2+i} - a_{N_2+i-1} \cdots a_{N_2} - a_{N_2-1} \cdots a_{N_2+i-n} - a_{N_2+i-n-1} \end{pmatrix} \begin{pmatrix} 1 \\ p_1 \\ \vdots \\ p_n \end{pmatrix}.$$

$$\because N_1 \geqslant n + 1 - i, \ \therefore N_1 + i - n - 1 \geqslant 0.$$

Using Lemma 1.1, $p_i^* = 0$.

The above theorem is the main result of this paper. Now we can dispose all or part of the coefficients of the closed-loop eigenpolynomial to zero. Considering the requirement of deadbeat control, the following theorem about GPC deadbeat control can be derived.

Theorem 3.2. *To the nth order plant, if $\lambda = 0$, $N_1 \geqslant n$, $N_u \geqslant n + 1$ and G is full rank in column, GPC results in deadbeat control, and the closed-loop system has the same dynamics.*

Proof. Under the conditions given above, for all $i = 1, 2, \cdots, n$, $N_u \geqslant n + 2 - i$, $N_1 \geqslant n + 1 - i$. Using Theorem 3.1, we have $p_1^* = \cdots = p_n^* = 0$. From Corollary 3.1, it follows $p_{n+1}^* = 0$.

We have $A_c(z^{-1})=1$, $G(z^{-1})=d_s z^{-1}B(z^{-1})$. So the GPC system becomes a deadbeat one. According to Lemma 1.3, $1+p_1^*+\cdots+p_{n+1}^*=d_sB(1)$. Since $p_1^*=\cdots=p_{n+1}^*=0$, then $d_s=1/B(1)$. $G(z^{-1})=z^{-1}B(z^{-1})/B(1)$. Therefore the closed-loop system has the same dynamics, independent of the choice of tuning parameters.

Furthermore, according to the solvability discussed above, the following theorem can be derived.

Corollary 3.2. *To the nth order plant, GPC results in deadbeat control if*
(i) $\lambda=0, N_u=n+1, N_1\geqslant n, N_2\geqslant N_1+n$ *or*
(ii) $\lambda=0, N_u\geqslant n+2, N_1=n, N_2\geqslant N_u+n-1$ *and* $m_n\neq 0$.

Comments: (i) Clarke and Mohtadi[5] proposed that GPC results in deadbeat control if the tuning parameters are chosen as $N_1=n^*, N_2\geqslant 2n^*-1, N_u=n^*$ and $\lambda=0$, where n^* should be $n+1$ because this result is based on the augmented state space model. It is obvious that Corollary 3.2 generalizes the conclusion in ref. [5] greatly.

(ii) Since the closed-loop transfer function is

$$G(z^{-1}) = (m_1z^{-1}+\cdots+m_nz^{-n})/\sum_{i=1}^{n} m_i,$$

the step response of the system can be exactly evaluated. Therefore, we derived not only the deadbeat control property but also the quantitative response of the deadbeat process.

4 Reduced-order property of GPC

From the mapping relationship in eq. (8) we know that the ordinary order of GPC closed-loop eigenpolynomial is $n+1$; that is, the closed-loop system has $n+1$ poles. Since the dynamic complexity of closed-loop system is determined by the location of poles, if some of them are disposed to the origin, they will not affect the dynamic process and the complexity of closed-loop system will be simplified. That refers to the reducing of the order of eigenpolynomial. Hence the study on reduced-order property of GPC is meaningful. The deadbeat control discussed above means that all of the poles are disposed to the origin, i.e. the eigenpolynomial is zero-ordered. It is the extreme case of reducing order. Now according to Theorem 3.1, we give the tuning parameters choosing for GPC order reducing in general.

Theorem 4.1. *To the nth order plant, if* $\lambda=0$, $N_2\geqslant N_1+N_u-1$, *the closed-loop GPC will have the order* n_c *given by*

$$n_c=\begin{cases} n-N_u+1, & 1\leqslant N_u\leqslant n, N_1\geqslant N_u-1,\\ n-N_1, & 1\leqslant N_1\leqslant n-1, N_u\geqslant N_1+1,\\ 0, & N_u\geqslant n+1, N_1\geqslant n.\end{cases}$$

Proof. First, according to Corollary 3.1, $p_{n+1}^*=0$. Then we will use Theorem 3.1 to prove the results:

(1) If $1\leqslant N_u\leqslant n$, $N_1\geqslant N_u-1$, then for all $n-N_u+2\leqslant i\leqslant n$, $N_u=n+2-(n-$

$N_u+2) \geqslant n+2-i$ and $N_1 \geqslant N_u-1 \geqslant n+1-i$. Using Theorem 3.1, $p^*_{n-N_u+2}=\cdots=p^*_n=0$, then $n_c=n-N_u+1$.

(2) If $1 \leqslant N_1 \leqslant n-1$, $N_u \geqslant N_1+1$, for all $n-N_1+1 \leqslant i \leqslant n$, $N_1=n+1-(n-N_1+1) \geqslant n+1-i$ and $N_u \geqslant N_1+1 \geqslant n+2-i$, so $p^*_{n-N_1+1}=\cdots=p^*_n=0$, then $n_c=n-N_1$.

(3) If $N_u \geqslant n+2$, $N_1 \geqslant n+1$, for all $1 \leqslant i \leqslant n$, using Theorem 3.1, $p_1^*=\cdots=p_n^*=0$. The proof is completed.

In Theorem 4.1, $N_2 \geqslant N_1+N_u-1$ is a necessary condition for G full rank in column and the choice of N_2 does not affect the order of the closed-loop system. In the theorem above, we also give the order of the closed-loop system in all the cases of N_1, N_u. Associating it with the solvability of control law, fig. 1 gives the graphic description of GPC order reducing. Note that the × in fig. 1 corresponds to the deadbeat control result in ref. [5] and the two zero lines to the results derived by Corollary 3.2 of this paper.

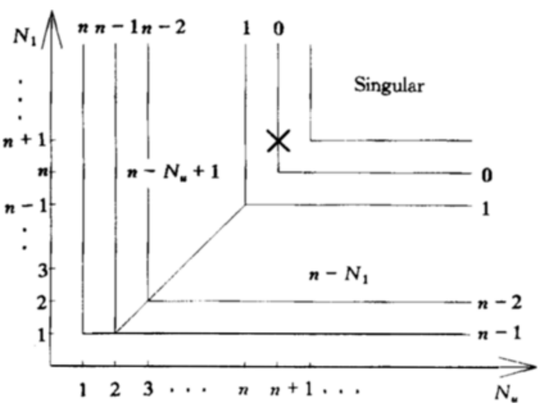

Fig. 1. The relationship between N_1, N_u and closed-loop order.

The condition for reducing the order of closed-loop GPC given in Theorem 4.1 is a new result which never appeared in literature. It is interesting and very useful for GPC system design and analysis. It also allows us to dispose the closed-loop order in $\{0, \cdots, n+1\}$ by choosing proper turning parameters. Especially, we can dispose the closed-loop GPC to first order.

Corollary 4.1. *To the nth order plant, if* $\lambda=0$ *and*

(i) $N_u=n$, $N_1 \geqslant n-1$, $N_2 \geqslant N_1+n-1$ *or*
(ii) $N_1=n-1$, $N_u \geqslant n$, $N_2 \geqslant N_u+n-2$,

the closed-loop GPC is first order with $A_c(z^{-1})=1+p_1^* z^{-1}$, *where* $p_1^*=d_sB(1)-1$.

Proof. According to Lemma 1.3, $1+p_1^*+\cdots+p^*_{n+1}=d_sB(1)$. Since $p_2^*=\cdots=p^*_{n+1}=0$, and $1+p_1^*=d_sB(1)$, $p_1^*=d_sB(1)-1$.

The above corollary gives a way to calculate p_1^* if the closed-loop GPC is first order under proper conditions. It is clear that the system stability can be checked immediately by determining whether $|p_1^*|<1$ and the closed-loop system transfer function is

$$G(z^{-1})=\frac{d_s z^{-1}B(z^{-1})}{A_c(z^{-1})}=\frac{d_s m_1 z^{-1}}{1+p_1^* z^{-1}}+\cdots+\frac{d_s m_n z^{-n}}{1+p_1^* z^{-1}}.$$

5 Conclusion

Explicit expression of GPC closed-loop eigenpolynomial is derived based on the IMC structure. Some theoretical results about the properties of closed-loop GPC system are discussed by studying the mapping relationship, and this also provides a new approach to study the properties of the predictive control systems.

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