INT. J. CONTROL, 1998, VOL. 70, NO. 5, 831–840

Brief Communication

Some GPC stability results

JUN ZHANG† and YUGENG XI†

This paper is concerned with stability problems in the original form of GPC. An explicit quantitative description of a GPC closed-loop system is presented. Some stability conditions for a closed-loop system and parameter design guidelines for plants with monotonic or convex step responses are derived.

1. Introduction

Model based predictive control (MBPC) has attracted much attention since the 1990s (see, for example, Garcia *et al.* (1989) and Richalet (1993)). The early MBPC algorithms, such as MAC (Richalet *et al.* 1978), DMC (Cutler and Ramaker 1980), EHAC (Ydstie 1984) and GPC (Clarke *et al.* 1987), are mainly based on input/output models. Each has a few different features which distinguish it from the others in the form of plant model, cost function, etc. Despite their popularity and success, they all suffer from a lack of clear theory dealing with closed-loop stability. A few theorems have been presented (Clarke and Mohtadi 1989, Scattolini and Bittanti 1990, Rawlings and Muske 1993) but the problem still remains inadequately examined.

In recent years, using fruitful results in state-space theory, some new types of MBPC algorithms, such as RHTC (Kwon and Byun 1989), SIORHC (Mosca and Zhang 1992), and CRHPC (Clarke and Scattolini 1991) have emerged. State-feedback control laws capable of stabilizing linear plants (Kleinman 1970, 1974, Kwon and Pearson 1975, 1978) have equipped these new algorithms with guaranteed stability properties under certain conditions. Simultaneously, extensions to basic MBPC schemes have also been developed with improved closed-loop stability (Demircioglu and Clarke 1993, Yoon and Clarke 1992).

Among the various kinds of MBPC methods, GPC can be regarded as a representative, for it is very general and is regarded as a superset of many others (Clarke and Mohtadi 1989). This paper deals with the GPC stability issue in its original form. Other than the methods presented above, we derived an explicit expression of a GPC closed-loop transfer function. This makes it possible to investigate the GPC stability directly via a closed-loop eigen polynomial. This work is organized as follows. Section 2 formulates the closed-loop description of GPC, which underlies the following theoretical research. In section 3, stability conditions for a GPC closed-loop system and limiting cases are studied. Section 4 outlines design guidelines for plants with monotonic or convex step responses parameters. Finally, our conclusions are given in section 5.

Received March 1996. Revised April 1997.

† Institute of Automation, Shanghai Jiao Tong University, Shanghai 200030, PR China, e-mail: ygxi@mail.sjtu.edu.cn.

2. Explicit closed-loop description of GPC

In traditional control theory, the eigen polynomial seems quite advantageous in system analysis. Some researchers have also formulated closed-loop transfer functions for MBPC to analyse their closed-loop properties (Maurath *et al.* 1988, Yoon 1994). However, the coefficients in these eigen polynomials are implicit so that they cannot be applied easily to system analysis. In this section, an explicit description of a GPC closed-loop system will be presented, which can be referred to as the coefficient mapping from plant to closed-loop system. It provides the basis of the research work in this paper.

GPC design is based on the following CARIMA model:

$$A(z^{-1})y(t) = B(z^{-1})u(t-1) + \xi(t)/\Delta, \quad \Delta = 1 - z^{-1} \tag{1}$$

where $u(t)$ is the control input, $y(t)$ the output, $\xi(t)$ an uncorrelated random sequence and A and B are polynomials in the backward shift operator z^{-1}:

$$B(z^{-1}) = b_1 + b_2 z^{-1} + \cdots + b_n z^{-n+1} \tag{2}$$

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_n z^{-n} \tag{3}$$

It is assumed that A and B are irreducible. This paper is not concerned with the case of dead-time since the conclusions here can be readily extended.

Consider the cost function:

$$J(t) = E\left\{ \sum_{j=N_1}^{N_2} \left[y(t+j) - \omega(t+j) \right]^2 + \sum_{j=1}^{N_u} \lambda \left[\Delta u(t+j-1) \right]^2 \right\} \tag{4}$$

here assume $\omega(t+j) = \omega$, $j \geq 0$ for simplicity.

The GPC control law is given by:

$$\Delta u(t) = d^{\mathrm{T}}(\mathbf{w} - \mathbf{f}) \tag{5}$$

where

$$\mathbf{w} = (\omega \quad \omega \quad \cdots \quad \omega)^{\mathrm{T}}$$

$$d^{\mathrm{T}} = (1 \quad 0 \quad \cdots \quad 0)(G^{\mathrm{T}} G + \lambda I)^{-1} G^{\mathrm{T}} \stackrel{\mathrm{def}}{=} (d_1 \quad \cdots \quad d_{N_2 - N_1 + 1}) \tag{6}$$

G is the matrix that consists of the plant's step responses $\{g_i\}$:

$$G = \begin{pmatrix} g_{N_1} & \cdots & g_1 & 0 & & \\ \vdots & & & \ddots & \ddots & \\ \vdots & & & & \ddots & 0 \\ g_{N_u} & \cdots & \cdots & \cdots & & g_1 \\ \vdots & & & & & \vdots \\ g_{N_2} & \cdots & \cdots & \cdots & & g_{N_2 - N_u + 1} \end{pmatrix}$$

In equation (5), $\mathbf{f}$ denotes the free responses of the plant:

$$\mathbf{f} = H\Delta u(t) + Fy(t) \tag{7}$$

$$H = \begin{bmatrix} z^{N_1-1}(G_{N_1} - g_{N_1}z^{-N_1+1} - \cdots - g_1) \\ \vdots \\ z^{N_2-1}(G_{N_2} - g_{N_2}z^{-N_2+1} - \cdots - g_1) \end{bmatrix} \tag{8}$$

$$F = (F_{N_1} \quad \cdots \quad F_{N_2})^{\mathrm{T}} \tag{9}$$

where $G_j = E_jB$, E_j and F_j satisfy the following Diophantine equation:

$$1 = E_j(z^{-1})A\Delta + z^{-j}F_j(z^{-1})$$

Lemma 1: *Let the plant be described by;*

$$G_{\mathrm{P}}(z^{-1}) = \frac{z^{-1}B(z^{-1})}{A(z^{-1})}$$

where A, B are given in (2) *and* (3). *Then the step responses* $\{g_i\}$ *and the coefficients of A, B satisfy:*

$$\begin{cases} g_1 = b_1 \\ (g_2 - g_1) + g_1a_1 = b_2 \\ \quad\vdots \\ (g_n - g_{n-1}) + (g_{n-1} - g_{n-2})a_1 + \cdots + g_1a_{n-1} = b_n \\ (g_{n+1} - g_n) + (g_n - g_{n-1})a_1 + \cdots + g_1a_n = 0 \\ (g_{i+2} - g_{i+1}) + (g_{i+1} - g_i)a_1 + \cdots + (g_{i-n+2} - g_{i-n+1})a_n = 0, \quad i \geq n \end{cases} \tag{10}$$

Proof: The relationship between the step responses and plant transfer function can be written as:

$$\frac{b_1z^{-1} + \cdots + b_nz^{-n}}{1 + a_1z^{-1} + \cdots + a_nz^{-n}} = g_1z^{-1} + (g_2 - g_1)z^{-2} + (g_3 - g_2)z^{-3} + \cdots$$

then the above recursion (10) follows. □

The above lemma reveals the relationship between step responses and coefficients of the plant transfer function. Especially, the last equation in (10) gives rise to a linear combination of a_i that has to be zero.

From (1), (5) and (7), we can easily derive the GPC transfer function:

$$G(z^{-1}) = \frac{d_{\mathrm{s}}z^{-1}B}{(1 + d^{\mathrm{T}}H)A\Delta + z^{-1}Bd^{\mathrm{T}}F} = \frac{d_{\mathrm{s}}z^{-1}B(z^{-1})}{A_c(z^{-1})} \tag{11}$$

where

$$d_{\mathrm{s}} = \sum_{i=1}^{N_2-N_1+1} d_i$$

Define

$$c_i = \sum_{j=1}^{N_2-N_1+1} d_j g_{i+j-1}$$

The dominator polynomial of $G_c(z^{-1})$ can be written as

$$\begin{aligned} A_c(z^{-1}) &= A\Delta(1 + c_{N_1+1}z^{-1} + c_{N_1+2}z^{-2} + \cdots) \\ &= A(z^{-1})(1 + (c_{N_1+1} - 1)z^{-1} + (c_{N_1+2} - c_{N_1+1})z^{-2} + \cdots) \end{aligned} \tag{12}$$

Let a_i^* denote the coefficients of $A_c(z^{-1})$, i.e.

$$A_c(z^{-1}) = 1 + a_1^* z^{-1} + a_2^* z^{-2} + \cdots$$

then a_i^* can be expressed as

$$\begin{bmatrix} 1 \\ a_1^* \\ \vdots \\ a_n^* \\ a_{n+1}^* \\ a_{n+2}^* \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 & & & 0 \\ c_{N_1+1} - 1 & 1 & & \\ \vdots & & \ddots & \\ c_{N_1+n} - c_{N_1+n-1} & \cdots & \cdots & 1 \\ c_{N_1+n+1} - c_{N_1+n} & \cdots & \cdots & c_{N_1+1} - 1 \\ c_{N_1+n+2} - c_{N_1+n+1} & \cdots & \cdots & c_{N_1+2} - c_{N_1+1} \\ \vdots & & & \vdots \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_n \end{bmatrix}$$

For all $i \geq 1$,

$$\begin{aligned} a_{n+i+1}^* &= \begin{bmatrix} c_{N_1+n+i+1} - c_{N_1+n+i} & \cdots & c_{N_1+i+1} - c_{n_1+i} \end{bmatrix} \mathbf{a} \\ &= \begin{bmatrix} d_1 & \cdots & d_{N_2-N_1+1} \end{bmatrix} \begin{pmatrix} g_{N_1+n+i+1} - g_{N_1+n+i} & \cdots & g_{N_1+i+1} - g_{N_1+i} \\ \vdots & & \vdots \\ g_{N_2+n+i+1} - g_{N_2+n+i} & \cdots & g_{N_2+i+1} - g_{N_2+i} \end{pmatrix} \begin{pmatrix} 1 \\ a_1 \\ \vdots \\ a_n \end{pmatrix} \end{aligned}$$

Using Lemma 1, it can be easily derived that $a^*_{n+i+1} = 0$, $i \geq 1$. Therefore,

$$A_c(z^{-1}) = 1 + a_1^* z^{-1} + \cdots + a_{n+1}^* z^{-(n+1)} \tag{13}$$

The coefficients of $A_c(z^{-1})$ are then determined by:

$$\begin{bmatrix} 1 \\ a_1^* \\ \vdots \\ a_{n+1}^* \end{bmatrix} = \begin{bmatrix} 1 & & & 0 \\ c_{N_1+1} - 1 & 1 & & \\ \vdots & \ddots & \ddots & 1 \\ c_{N_1+n+1} - c_{N_1+n} & \cdots & \cdots & c_{N_1+1} - 1 \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} \tag{14}$$

It can be found that the minimal DMC controller (Xi 1989) has the same eigen polynomial with the coefficients given by equation (14). Then the conclusions in this paper are also applicable to DMC and some other MBPC algorithms. Obviously, the closed-loop stability only depends upon the location of the roots of $A_c(z^{-1})$. So we will focus on equation (14) which shows the coefficient mapping from the eigen polynomial of the plant to that of the GPC closed-loop system.

3. Study of necessary conditions and limiting cases

Since the closed-loop dynamics are determined by the eigen polynomial $A_c(z^{-1})$, we could study the stability of GPC by analysing the coefficient mapping relationship (14).

Recall that, given a polynomial

$$f(z^{-1}) = 1 + f_1 z^{-1} + \cdots + f_n z^{-n} \tag{15}$$

a necessary condition of $f(z^{-1})$ to be stable is $f(1) > 0$, Then we could derive a necessary condition for GPC stability.

Theorem 1: *A necessary condition for GPC stability is $d_s B(1) > 0$.*

Proof: Since a necessary condition for $A_c(z^{-1})$ to be stable is $A_c(1) > 0$, i.e.

$$1 + a_1^* + \cdots + a_{n+1}^* > 0$$

multiply $(1 \quad \cdots \quad 1)$ with both sides of equation (14):

$$1 + a_1^* + \cdots + a_{n+1}^* = (c_{N_1+n+1} \quad c_{N_1+n} \quad \cdots \quad c_{N_1+1})\mathbf{a}$$

$$= (d_1 \quad \cdots \quad d_{N_2-N_1+1}) \begin{pmatrix} g_{N_1+n+1} & g_{N_1+n} & \cdots & g_{N_1+1} \\ \vdots & \vdots & & \vdots \\ g_{N_2+n+1} & g_{N_2+n} & \cdots & g_{N_2+1} \end{pmatrix} \begin{pmatrix} 1 \\ a_1 \\ \vdots \\ a_n \end{pmatrix}$$

Add the first $n + i$-1 equations of (10) together,

$$g_{i+n-1} + g_{i+n-2} a_1 + \cdots + g_{i-1} a_n = b_1 + \cdots + b_n = B(1). \quad i \geq 1$$

so

$$1 + a_1^* + \cdots + a_{n+1}^* = d_s B(1)$$

then $d_s B(1) > 0$ is a necessary condition for GPC closed-loop stability. □

For a stable plant, it holds that

$$g_\infty = B(1)/A(1)$$

and $A(1) > 0$, so that the condition in Theorem 1 will be $d_s g_\infty > 0$. This suggests that the GPC system is stable only when d_s has the same sign as the plant gain. Furthermore, we have the following corollary.

Corollary 1: *For a stable plant, a necessary condition for stabilizing GPC, by increasing λ, is*

$$\left(\sum_{i=N_1}^{N_2} g_i \right) g_\infty > 0$$

Proof:

$$d^{\mathrm{T}} = (1 \quad 0 \quad \cdots \quad 0)(G^{\mathrm{T}}G + \lambda I)^{-1}G^{\mathrm{T}}$$
$$= \frac{1}{\lambda}(1 \quad 0 \quad \cdots \quad 0)\left(\frac{1}{\lambda}G^{\mathrm{T}}G + I\right)^{-1} G^{\mathrm{T}}$$
$$\to \frac{1}{\lambda}\left[g_{N_1} \quad \cdots \quad g_{N_2}\right] \quad \text{as } \lambda \to \infty$$

so

$$d_s g_\infty \to \frac{1}{\lambda}\left(\sum_{i=N_i}^{N_2} g_i\right) g_\infty, \quad \text{as } \lambda \to \infty$$

This completes the proof. □

Corollary 1 points out that, even for stable plant, it is not always possible to stabilize its GPC system by increasing λ. It is required to select the predict horizon $[N_1 \quad N_2]$ properly so that the average step response over this horizon has the same sign as the plant gain.

Now limiting cases will be considered.

Theorem 2: *Assume that the system is open-loop stable. If $N_u = 1$, $\lambda = 0$, then there exists N_0 such that the GPC system is stable for all $N_2 \geq N_0$.*

Proof: For $N_u = 1$ and $\lambda = 0$,

$$d^{\mathrm{T}} = (g_{N_1} \quad \cdots \quad g_{N_2}) \Big/ \sum_{j=N_1}^{N_2} g_j^2$$

Then

$$c_i = \sum_{j=1}^{N_2 - N_1 + 1} g_{N_1+j-1} g_{i+j-1} \Big/ \sum_{j=N_1}^{N_2} g_j^2$$

It is not difficult to obtain $c_i \to 1$, as $N_2 \to +\infty$ and substituting this into (14) yields:

$$\begin{pmatrix} 1 \\ a_1^* \\ \vdots \\ a_{n+1}^* \end{pmatrix} = \begin{pmatrix} 1 & & & 0 \\ 0 & 1 & & \\ \vdots & & \ddots & \\ \vdots & & & 1 \\ 0 & \cdots & \cdots & 0 \end{pmatrix} \begin{pmatrix} 1 \\ a_1 \\ \vdots \\ a_n \end{pmatrix}$$

From the mapping relationship above, it is apparent that $A_c(z^{-1}) = A(z^{-1})$. So when $N_2 \to +\infty$, $A_c(z^{-1})$ is stable. Then there exists N_0 such that for $N_2 \geq N_0$, the closed-loop system is stable. □

Theorem 3: *For a stable plant, if $N_u = 1$, then there exists N_0, such that the closed-loop system is stable when $N_2 - N_1 \geq N_0$.*

Proof: The proof is similar to that of Theorem 2. □

4. Stable tuning parameters design

Though limiting cases are not difficult to study, the whole GPC stability problem involves considerable difficulties. The GPC closed-loop description derived above allows direct determination of whether a particular system is stable. Moreover, it can be utilized to analyse the general GPC stability problem.

Consider the following well known sufficient conditions of $f(z^{-1})$ to be stable:

(1) $1 > \sum_{i=1}^{n} |f_i|$ or

(2) $1 > f_1 > \cdots > f_n > 0$

Equation (12) can then be written as

$$\frac{A_c(z^{-1})}{A(z^{-1})} = 1 + (c_{N_1+1} - 1)z^{-1} + (c_{N_1+2} - c_{N_1+1})z^{-2} + \cdots \tag{16}$$

For a stable plant, all the roots of $A(z^{-1})$ are located in the unit circle. If equation (16) does not equal zero for $|z| \geq 1$, then $A_c(z^{-1})$ is stable, i.e. the closed-loop system is stable. Using Hurwitz's Theorem in complex analysis it can be proven that, given

$$f(z^{-1}) = 1 + f_1 z^{-1} + f_2 z^{-2} + \cdots$$

if

(1) $1 > \sum_{i=1}^{\infty} |f_i|$ or (17)

(2) $1 > f_1 > f_n > \cdots > 0$ (18)

then $f(z^{-1})$ will never equal zero when $|z| \geq 1$, i.e. $f(z^{-1})$ is stable.

Using the above sufficient conditions, we can discuss GPC stability based on the property of step responses.

Theorem 4 (Yoon 1994): *Assume the stable plant has a monotonic step response after k, i.e.*

$$0 \leq g_k \leq g_{k+1} \leq \cdots \leq g_\infty$$

then any selection of $N_2 \geq N_0$ and $N_1 \geq k$, $N_u = 1$, $\lambda = 0$ results in closed-loop stability, where N_0 satisfies

$$\sum_{i=N_1}^{N_0} g_i^2 > \frac{g_\infty}{2} \sum_{i=N_1}^{N_0} g_i$$

Then the following corollary is straightforward.

Corollary 2: *For the plant given in Theorem 4, $N_1 \geq k$, $N_u = 1$ and $\lambda = 0$, any N_2 satisfying:*

$$\frac{1}{N_2 - N_1 + 1} \sum_{i=N_1}^{N_2} g_i > \frac{g_\infty}{2} > 0$$

results in closed-loop stability.

Proof: If the above condition holds, since

$$\left(\frac{1}{N_2 - N_1 + 1} \sum_{i=N_1}^{N_2} g_i^2 \right)^{1/2} \geq \frac{1}{N_2 - N_1 + 1} \sum_{i=N_1}^{N_2} g_i$$

then

$$\sum_{i=N_1}^{N_2} g_i^2 \geq \frac{1}{N_2 - N_1 + 1} \left(\sum_{i=N_1}^{N_2} g_i \right)^2 \geq \frac{g_\infty}{2} \sum_{i=N_1}^{N_2} g_i$$

From Theorem 4, the closed-loop system is stable. □

The above design condition can be simplified further.

Corollary 3: *For the plant given in Theorem 4, if $N_1 \geq k$, $N_u = 1$, $\lambda = 0$ and N_1 satisfies*

$$g_{N_1} > \frac{g_\infty}{2} > 0$$

then the closed-loop system is stable.

Proof: From the monotonicity of $\{g_i\}$, $g_{N_2} \geq \cdots \geq g_{N_1} \geq 0$ it follows that

$$\frac{1}{N_2 - N_1 + 1} \sum_{i=N_1}^{N_2} g_i \geq g_{N_1} > \frac{g_\infty}{2} > 0$$

Using Corollary 2, the desired result can be derived. □

These two corollaries have practical meaning. For example, Corollary 2 indicates that, for the monotonic case, closed-loop stability is guaranteed when the average of the step responses in the prediction horizon is greater than half the plant grain.

For a plant with a convex step response, the following theorem can be given.

Theorem 5: *Assume the stable plant has a convex step response after k,* i.e.

$$g_k > g_{k+1} - g_k > g_{k+2} - g_{k+1} > \cdots > 0$$

then any $N_1 \geq k$, $N_u = 1$ and $\lambda = 0$ results in closed-loop stability.

Proof: Let

$$s = 1 \Big/ \sum_{i=N_1}^{N_2} g_i^2$$

and $c_{N_1} = 1$ holds, so

$$1 = s\left[g_{N_1}^2 + \cdots + g_{N_2}^2\right] > 0$$

$$c_{N_1+1} - 1 = s\left[g_{N_1}(g_{N_1+1} - g_{N_1}) + \cdots + g_{N_2}(g_{N_2+1} - g_{N_2})\right] > 0$$

$$c_{N_1+2} - c_{N_1+1} = s\left[g_{N_1}(g_{N_1+2} - g_{N_1+1}) + \cdots + g_{N_2}(g_{N_2+2} - g_{N_2+1})\right] > 0$$

$$\vdots$$

Since $N_1 \geq k$, then

$$g_j > g_{j+1} - g_j > g_{j+2} - g_{j+1} > \cdots > 0, \quad j \geq N_1$$

so

$$1 > c_{N_1+1} - 1 > c_{N_1+2} - c_{N_1+1} > \cdots > 0$$

From the sufficient condition in (18), the closed-loop system will be stable. □

Comments

- For the plant in which $g_i < 0$, similar results can also be obtained.
- Theorem 4 or 5 only requires the monotonic or convex condition after a certain time, not from the beginning, so these theorems are valid for a non-minimum phase plant.

Theorem 6: *For an nth order plant, if*

(1) $N_u = n + 1$, $N_1 \geq n$, $N_2 \geq N_1 + n$ *or*
(2) $N_u \geq n + 2$, $N_1 = n$, $N_2 \geq N_u + n - 1$, $b_n \neq 0$

then there exists a constant $\lambda_0 > 0$ *such that, for* $\lambda \leq \lambda_0$, *the closed-loop GPC is stable.*

Proof: The poles of the closed-loop GPC vary continuously with the parameter λ. Under the condition above and $\lambda = 0$, the closed-loop system will result in deadbeat control (Zhang 1996). That is, all the closed-loop poles are placed at the origin. As λ is increased, they leave the origin, but an upper bound of λ exists, say λ_0, so that for $\lambda \leq \lambda_0$, all the closed-loop poles are inside the unit circle and the closed-loop system is stable. □

Though we know the existence of λ_0 in the above theorem, the exact value of λ_0 still remains unknown. Further investigation is required to find this upper bound.

5. Conclusion

This work has elaborated on a new analysis technique for GPC which is known as coefficient mapping from a plant to a closed-loop system. With this technique we have performed stability studies of a GPC system. Some stability conditions of a closed-loop system and design guidelines for plants with monotonic or convex step responses have been derived. It has been shown that coefficient mapping is an effective method of analysing GPC closed-loop properties.

Acknowledgments

This work was supported by the National Natural Science Foundation of China.

References

Clarke, D. W., and Mohtadi, C., 1989, Property of generalized predictive control. *Automatica,* **25**, 859–875.

Clarke, D. W., and Scattolini, R., 1991, Constrained receding horizon predictive control. *IEE Proceedings* D, **138**, 347–354.

Clarke, D. W., Mohtadi, C., and Tuffs, P. S., 1987, Generalized predictive control. Parts 1 and 2. *Automatica,* **23**, 137–160.

Cutler, C. R., and Ramaker, B. L., 1980, Dynamic matrix control — a computer control algorithm. *Proceedings of the Joint Automatic Control Conference,* San Francisco, USA, WP5-B.

Demircioglu, H., and Clarke, D. W., 1993, Generalized predictive control with end-point state weighting. *IEE Proceedings* D, **140**, 275–292.

Garcia, C. E., Prett, D. M., and Morari, M., 1989, Model predictive control: theory and practice — a survey. *Automatica*, **25**, 335–348.

Kleinman, D. L., 1970, An easy way to stabilize a linear constant system. *IEEE Trans. Automat. Contr.,* **15**, 692; 1974, Stabilizing a discrete, constant, linear system with application to iterative method for solving the Riccati equation. *IEEE Trans. Automat. Contr.,* **19**, 252–254.

Kwon, W. H., and Byun, D. G., 1989, Receding horizon tracking control as a predictive control and its stability properties. *Int. J. Contr.,* **50**, 1807–1824.

Kwon, W. H., and Pearson, A. E., 1975, On the stabilization of a discrete constant linear system. *IEEE Trans. Automat. Contr.,* **20**, 800–801; 1978, On feedback stabilization of time-varying discrete linear systems. *IEEE Trans. Automat. Contr.,* **23**, 479–481.

Maurath, P. R., Mellichamp, D. A., and Seborg, D. E., 1988, Predictive controller design for single-input/single output (SISO) systems. *Ind. Eng. Chem. Res.,* **27**, 956–963.

Mosca, E., and Zhang, J. X., 1992, Stable redesign of predictive control. *Automatica,* **28**, 1229–1233.

Rawlings, J. B., and Muske, K. B., 1993, The stability of constrained receding horizon control, *IEEE Trans. Automat. Contr.,* **38**, 1512–1516.

Richalet, J., 1993, Industrial applications of model based predictive control. *Automatica,* **29**, 1251–1274.

Richalet, J., Rault, A., Testud, J. L., and Papon, J., 1978, Model predictive heuristic control: applications to industrial processes. *Automatica,* **14**, 413–428.

Scattolini, R., and Bittani, S., 1990, On the choice of the horizon in long-range predictive control — some simple criteria. *Automatica,* **26**, 915–917.

Xi, Y. G., 1989, Minimal form of a predictive controller based on the step-response model. *Int. J. Contr,* **49**, 57–64.

Ydstie, B. E., 1984, Extended horizon adaptive control. *Proceedings of the IFAC World Congress,* Budapest, Hungary, pp. 133–137.

Yoon, T. W., 1994, Robust adaptive predictive control. Ph. D. Thesis, Dept. Eng. Sci., Oxford University, UK.

Yoon, T. W., and Clarke, D. W., 1993, Receding horizon predictive control with exponential weighting. *Int. J. Syst. Sci.,* **24**, 1745–1757.

Zhang, J., 1996, Property analysis of GPC based on coefficient mapping. *Proceedings of the IFAC World Congress,* San Francisco, USA, pp. 457–462.